Textbook reading for this week:

- §2.4 (applications of rank-nullity)
- §2.5 (composition and matrix multiplication)
- §2.6 (inverse transformations)
- §2.7 (change of basis)

Study items:

- Know the statement of the Rank-Nullity theorem, and the explanation via counting pivot and non-pivot columns.
- How you use dimension to determine whether a transformation is injective and/or surjective?
- What does it mean for a linear transformation to be invertible? How do you determine whether it is invertible by counting dimensions?
- What does it mean to say that two vector spaces are isomorphic?
- Explain why V is isomorphic to $\mathbb{R}^n$, if n is the dimension of V.
- Know the "two out of three theorems," for determining when a set is a basis, and for determining when a transformation is invertible. How do you use this theorem to more quickly check that T is both injective and surjective?

Note. The items below were covered after we finished midterm 2 material, and therefore may appear on midterm 3.

- Definition: the composition of two linear transformations.
- How do you compute the matrix representation of a composition of linear transformations?
- Be able to multiply matrices when compatible.

Problems:

1. *(Damiano–Little 2.4.2(b,c,d,e))* (Determining whether a transformation with given matrix representation is injective, surjective, or both)
2. *(Damiano–Little 2.4.4(d,e))* (linear systems in matrix form)
3. *(Damiano–Little 2.5.2)* (distributivity of composition)
4. *(Damiano–Little 2.5.5(b,c,d,e))* (matrix multiplication examples)
5. *(Damiano–Little 2.5.8(a))* (matrix representation of derivatives and integrals)
6. *(Damiano–Little 2.5.12(a))* (are injectivity and surjectivity preserved under composition?)
7. Suppose $S : U \to V$ is a surjective linear transformation and $T : V \to W$ is any linear transformation. Prove that $\mathrm{Im}(TS) = \mathrm{Im}(T)$.
8. *(Damiano–Little 2.6.1(a,b))* (Determine whether a transformation is invertible, and find the inverse if so)
9. *(Damiano–Little 2.6.5)* (formula $(AB)^{-1} = B^{-1}A^{-1}$)