Textbook reading for this week:

- §2.7 (change of basis)
- The following key points from §3. You can omit the proofs (though of course there is value in them if you are interested).
  - Exposition on pp. 133-136 and 140-141 on geometric interpretation of determinant and the notion of an alternating multilinear functions.
  - Definition 3.2.4 on p. 142 (the ijth minor of a matrix)
  - Theorems/Lemmas/Propositions (not their proofs): 3.2.8, 3.2.9, 3.2.12, 3.2.14, 3.3.4, 3.3.7, 3.3.8
  - Examples 3.2.9 (p. 148) and 3.2.15 (p. 150) on the two main ways to compute larger determinants.

Study items:

- Definition: the composition of two linear transformations.
- How do you compute the matrix representation of a composition of linear transformations?
- Be able to multiply matrices when compatible.
- Compute the change-of-basis matrix for two different bases of the same vector space.
- Use the change-of-basis formula to compute the matrix representation of a linear transformation.
- The formulas for 2×2 and 3×3 determinants.
- The geometric meaning of determinant, in terms of volume expansion.

Problems:

1. Let $T : V \to W$ be an invertible linear transformation, and let α, β be bases for V, W, respectively. Prove that
$$[T]_\alpha^\beta [T^{-1}]_\beta^\alpha = I_n,$$
where $n = \dim V$. Conclude that $\left([T]_\alpha^\beta\right)^{-1} = [T^{-1}]_\beta^\alpha$.
2. *(Damiano–Little 2.6.3(a,e,g))* (Determine if a matrix is invertible, and find the inverse if so)
3. *(Damiano–Little 2.7.2)* (determine matrix in standard basis given some values)
4. *(Damiano–Little 2.7.4(b,c))* (find representation in standard basis from representation in another basis)
5. *(Damiano–Little Chapter 2 supplementary (pp. 130-132), #3)*
6. *(Damiano–Little Chapter 2 supplementary (pp. 130-132), #5)*
7. *(Damiano–Little 3.1.3)* (2×2 determinants)

8. *(Damiano–Little 3.1.4)* (determinant of a rotation matrix)

Note. The following two questions concern Manuel González Villa's guest class on Friday, April 22. You will not be required to understand all content from that presentation in complete detail, but we will likely discuss more details about it soon. The questions below are meant to encourage you to reflect on the material and tell me of anything interesting you'd like me to further explain at the end of the course. You will receive full points for a good faith effort.

9. List at least one term you learned in the presentation that you don't know and/or you would like to known more about, and at least one question you have about principal components analysis and/or dimensionality reduction.

10. Suppose that you conduct m experiments, and in each experiment you measure n variables. You may view your measurements as m vectors in $\mathbb{R}^n$. In the following two questions, you can answer in words and do not need to be completely precise.

    (a) Explain why it may be practically useful to choose a different basis for $\mathbb{R}^n$ to use when studying these data. What characteristics might you look for in a good choice of basis for this purpose?

    (b) Suppose that, in order to decrease the number of dimensions, you consider only the coordinates corresponding to the first several basis vectors in your new basis. How might you decide to put the basis vectors in order so that you can reduce dimensions while retaining as much useful informaton as possible about your data set?