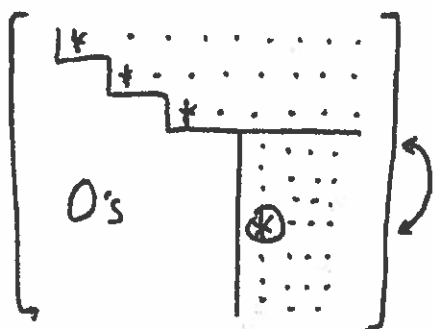
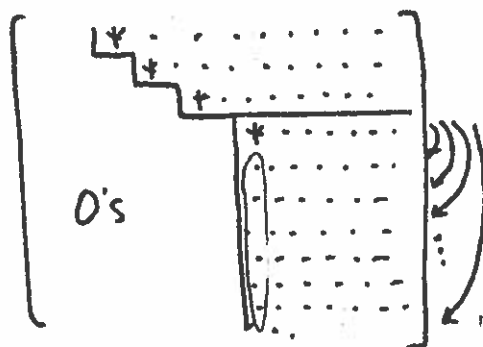


Reducing a matrix to row echelon form (REF):



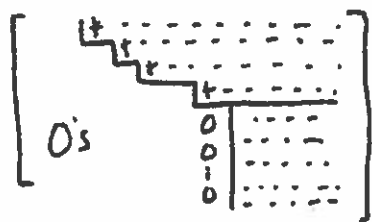
*'s are pivots
 0's might be zero or nonzero.

① swap two rows to put your next pivot in the row you want it. (if necessary/desired)



② Subtract multiples of that row from the others to make entries below the pivot 0.

want to "zero out" the entries below.



③ Repeat, until you are in REF.

example

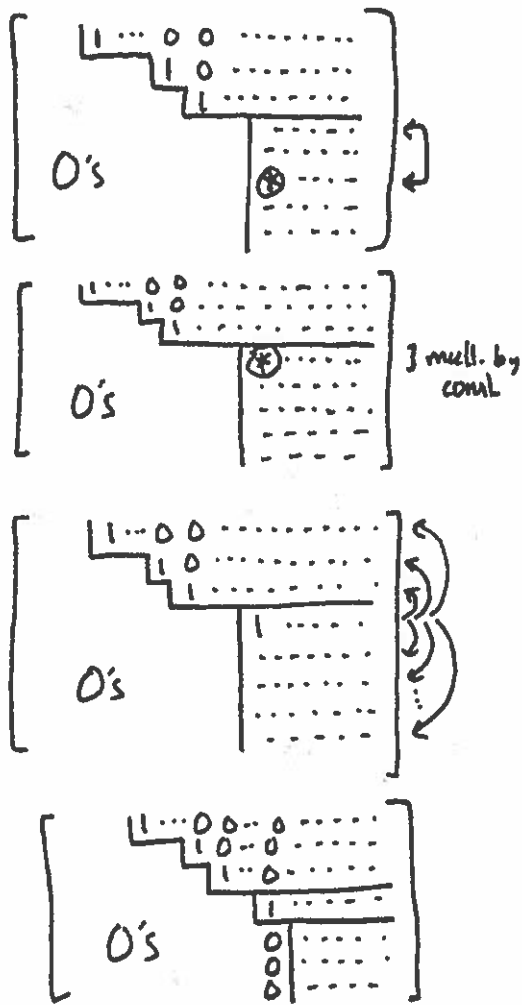
$$\begin{bmatrix} 0 & 1 & 2 & 5 \\ \textcircled{3} & 3 & 3 & 18 \\ 3 & 5 & 9 & 30 \end{bmatrix} \rightarrow \begin{bmatrix} \textcircled{3} & 3 & 3 & 18 \\ 0 & 1 & 2 & 5 \\ 3 & 5 & 9 & 30 \\ -3 & -3 & -3 & -18 \end{bmatrix} \xrightarrow{-1x} \begin{bmatrix} 3 & 3 & 3 & 18 \\ 0 & \textcircled{1} & 2 & 5 \\ 0 & 2 & 6 & 12 \\ -2 & -4 & -10 & -10 \end{bmatrix} \xrightarrow{-2x}$$

$$\begin{bmatrix} 3 & 3 & 3 & 18 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

now echelon form!

Reducing a matrix to reduced row echelon form

Note: many ways/choices!



- ① Swap two rows to put your next pivot where you want it.
(if necessary/desired)
- ② Multiply the row by the reciprocal of the pivot, so that it becomes 1.
- ③ Subtract multiples of that row from the others to make entries below & above the pivot 0.
- ④ Repeat until you're in RREF.

example

$$\begin{bmatrix} 0 & 1 & 2 & 5 \\ 3 & 3 & 3 & 18 \\ 3 & 5 & 9 & 30 \end{bmatrix} \xrightarrow{\text{row 2} \times \frac{1}{3}} \begin{bmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 6 \\ 3 & 5 & 9 & 30 \end{bmatrix} \xrightarrow{\text{row 1} \leftrightarrow \text{row 2}} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 3 & 5 & 9 & 30 \end{bmatrix} \xrightarrow{\text{row 3} - 3 \times \text{row 2}} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 2 & 6 & 12 \end{bmatrix} \xrightarrow{\text{row 3} - 2 \times \text{row 2}} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 2 & 2 \end{bmatrix} \xrightarrow{\text{row 3} \times \frac{1}{2}} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\text{row 1} - \text{row 2}} \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\text{row 1} + \text{row 3}} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\text{row 2} - 2 \times \text{row 3}} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

reduced row echelon form!