Study guide

- (§1.5) Understand how to convert a linear system of equations to a matrix equation ($A\vec{x} = \vec{b}$) and vice versa.
- (§1.4) Know the defintions of the identity matrix I_n (or I, when n is clear from context) and the inverse matrix A^{-1} (when it exists).
- (§1.4) Remember: only square matrices (same number of rows as number of columns) can possibly be invertible.
- (§1.4) Be able to quickly invert 2×2 matrices, e.g. using the formula mentioned in class.
- (§1.4) Be able to invert matrices of any size, and to tell when the inverse doesn't exist, by row-reducing a large matrix.
- (§1.5) Know how to use the inverse matrix (if it exists) to solve a matrix equation.

Textbook problems

- §1.4: 2, 6, 8, 11, 22
- §1.5: 14, 18, 30

Supplemental problems:

1. (a) Suppose that A is an invertible $n \times n$ matrix. Prove that any matrix equation $A\vec{x} = \vec{b}$ (where $\vec{b}$ is known and $\vec{x}$ is unknown) has a *unique* solution.

   (b) Suppose that A is a matrix with more columns than rows (i.e. A is $m \times n$, where $m < n$). Prove that any matrix equation $A\vec{x} = \vec{b}$ is either inconsistent or has infinitely many solutions. (*Hint:* after A is reduced to RREF, how many pivots can there be? What does this say about the number of free variables?)

   Comment: the problem above gives one way to see why a matrix with more columns that rows cannot possibly be invertible.

2. Suppose that A is an invertible matrix. Prove that the transpose A^t is also invertible, and that its inverse is given by $(A^{-1})^t$.

   Hint: You may wish to make use of some algebraic facts about transposition, summarized in Theorem 6 on page 36 of the textbook.

   Comment: Combining the two problems above shows that a matrix with more rows and columns cannot be invertible either (since its transpose has more columns than rows). Therefore this gives one way to prove that only square matrices can possibly inverses.