**Note:** this problem set will cover two weeks of material, and will therefore be longer than usual. This is because there will not be an assignment due on the day of the midterm.

**Study guide**

- (§2.1 and 2.2) Know the definition of "linear combination," and visual interpretation.
- (§2.2) How can you tell whether a particular vector is a linear combination of some other vectors? How can you desribe the set of all vectors that can be written as a linear combination of a specific set of vectors?
- (§2.2) Understand how to interpret a matrix equation $A\vec{x} = \vec{b}$ in terms of linear combinations of the columns of $A$.
- (§2.3) Know and understand the formal definitions of linear dependence and linear independence.

  **Midterm 1 material ends here.**
- (§2.3) Understand how linear dependence tells you about redundency in a list of vectors.
- (§2.3) How do you prove that a list of vectors is linearly independent? Be comfortable with the proof template from class about this.
- (§2.3) If you already know that a list of vectors is linearly independent, how do you make use of this fact in a proof?
- (§6.1) Know the defintion of the dot product as a sum of products of numbers.
- (§6.1) Know the geometric formula for dot product (in terms of $\cos\theta$). (You do not need to know the proof)
- (§6.1) How can you use dot products to measure *lengths* and *angles*?

**Textbook problems**

- §2.1: 18, 30
- §2.2: 2, 12, 32, 33, 38
- §2.3: 6, 8, 37, 42

  (*Note* about the wording in 2.3.42: This is two problems in one: first prove the claim stated in the second sentence, about $n \times n$ invertible matrices. The last sentence is asking you to show that the claim is *false* if you don't assume that $A$ is invertible; you should write down a *specific* $2 \times 2$ matrix and two linearly independent vectors $\vec{w}_1, \vec{w}_2$ such that $\{A\vec{w}_1, A\vec{w}_2\}$ is linearly dependent.)

  **Midterm 1 material ends here**
- §6.1: 14, 18, 23, 32

**Supplemental problems:**

1. Suppose that $\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$ is a linearly independent set of vectors in $\mathbb{R}^n$, and let $\vec{v}_{n+1}$ be some other vector in $\mathbb{R}^n$. Prove that $\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n, \vec{v}_{n+1}\}$ is linearly dependent *if and only if* $\vec{v}_{n+1}$ is a linear combination of the vectors $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$.

   *Note:* be very careful in your proof that you never assume that a number is nonzero without justifying why it must be so.

   **Midterm 1 material ends here**

2. A square matrix $A$ is called *orthogonal* if it is invertible and $A^{-1} = A^t$. (Note: read Theorem 15 in §1.6 of the book. We did not explicitly mention all of these facts in class, but a couple of them are useful here.)

   (a) Prove that if $A$ is orthogonal, then $\det A = \pm 1$.

   (b) Prove that if $A$ is an orthogonal $n \times n$ matrix and $\vec{v}$ is any vector in $\mathbb{R}^n$, then $\|A\vec{v}\| = \|\vec{v}\|$.

   (c) Prove that if $A$ is an orthogonal $n \times n$ matrix and $\vec{u}, \vec{v}$ are any two nonzero vectors in $\mathbb{R}^n$, then the angle between $A\vec{u}$ and $A\vec{v}$ is the same as the angle between $\vec{u}$ and $\vec{v}$.

   (d) Prove that the product of two orthogonal matrices of the same size is orthogonal, and that the inverse of an orthogonal matrix is orthogonal.

   *Note:* Orthogonal matrices arise in physics and engineering, because they represent rigid motions (rotations, etc.), which is demonstrated by the properties discussed above.