Study guide

- Know the definition of an *orthogonal basis* and of an *orthonormal basis.*
- (§6.2) Know the definition of an *inner product*, and some examples. Also know the terminology *inner product space* (see terminology note below)
- (§6.2) How do you define $\|\vec{v}\|$ and $\vec{u} \perp \vec{v}$ in an inner product space (not necessarily $\mathbb{R}^n$)? (See terminology notes below)
- (Friday's lab; see also §6.3)) Understand the *Gram-Schmidt procedure* for turning a basis $B = \{\vec{v}_1, \cdots, \vec{v}_n\}$ into an *orthonormal* basis $\{\vec{u}_1, \cdots, \vec{u}_n\}$.
- (§6.3) Know the definition of the projection $\text{proj}_{\vec{v}}\vec{u}$ in an inner product space, as well as the interpretation as the "nearest multiple" of $\vec{v}$ to $\vec{u}$.

Terminology notes An *inner product space* refers to a vector space with a chosen inner product. The problems below use some vocabulary that we've defined in the context of *dot products*, but is now being used in the more general setting of *inner products.* We haven't used it all in class in the more general setting yet, so here's a quick summary: the *norm* of a vector in an inner product space is $\vec{u}$ is $\|\vec{u}\| = \sqrt{\langle \vec{u}, \vec{u} \rangle}$. Two vectors $\vec{u}, \vec{v}$ are *orthogonal* if $\langle \vec{u}, \vec{v} \rangle = 0$; we also write $\vec{u} \perp \vec{v}$ as shorthand. A *set* of vectors is orthogonal if any two of them are orthogonal. The *angle* between two vectors $\vec{u}, \vec{v}$ is defined to be $\arccos\left(\frac{\langle \vec{u}, \vec{v} \rangle}{\|\vec{u}\| \cdot \|\vec{v}\|}\right)$.

Textbook problems

Notes about terminology: the problems below use some terminology and notation that we have defined in the context of dot products

- §6.2: 2, 4, 10, 12, 29(a,b,c)

  (in these first three problems: either check the three axioms of an inner product, or show that one of them fails)

- §6.3: 10, 18 (You may want to wait until after Friday's lab to think about these)

Supplemental problems:

1. Let V be an inner product space. Prove that if $S = \{\vec{v}_1, \cdots, \vec{v}_n\}$ is a list of vectors, and $\vec{u}$ is another vector such that $\vec{u} \perp \vec{v}_i$ for all i, then $\vec{u}$ is also orthogonal to any linear combination of the elements of S.

2. Prove that if $\vec{u}, \vec{v}$ are orthogonal vectors in an inner product space, then $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2+\|\vec{v}\|^2$. (This is the "Pythagorean theorem for inner product spaces.")