1. [9 points] Solve the following system of linear equations.

$$\begin{array}{rcrcrcrcr} & & x_2 & + & x_3 & + & x_4 & = & 5 \\ x_1 & & & + & 3x_3 & + & 7x_4 & = & 14 \\ x_1 & & & + & 2x_3 & + & 5x_4 & = & 11 \end{array}$$

2. [9 points] Recall that two matrices A, B *commute* if $AB = BA$. Consider the following three matrices.

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$$

(a) Determine whether A and B commute. (c) Determine whether A and C commute.

(b) Determine whether B and C commute.

3. [9 points] (a) Suppose that A is an $n \times n$ matrix. Show that if A is invertible, then $A\vec{x} = \vec{0}$ has no nontrivial solutions $\vec{x}$.

(b) Using part (a), show that $A = \begin{pmatrix} 1 & 1 & -2 \\ 2 & -3 & 1 \\ -1 & -1 & 2 \end{pmatrix}$ is not an invertible matrix. (*Hint:* the numbers in each row sum to 0.)

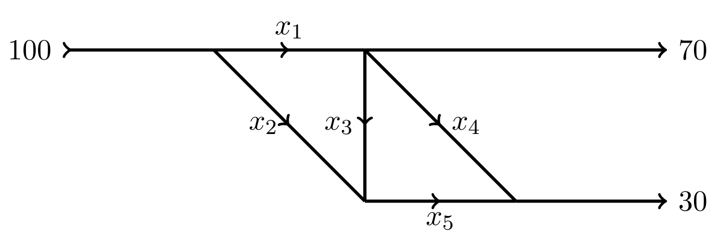
4. [9 points] The augmented matrix of a linear system has the form

$$\begin{pmatrix} 1 & 2 & -1 & a \\ 2 & 3 & -2 & b \\ -1 & -1 & 1 & c \end{pmatrix}.$$

(a) Determine the values of a, b, c for which this linear system in consistent.

(b) For those values of a, b, c for which the system is consistent, does it have a unique solution or infinitely many solutions? Briefly explain why.

5. [9 points] Write a system of linear equations that describes the traffic flow pattern for the network in the figure. **You do not need to solve the system.**



6. [9 points] Consider the following three vectors.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

(a) Show that the only choice of constants c_1, c_2, c_3 such that $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$ is $c_1 = c_2 = c_3 = 0$.

(b) Find scalars (constants) c_1, c_2, c_3 such that the vector

$$\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

is equal to $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$.