math 272
1/23/18

Encoding linear systems in "augmented matrices"

This saves some repetitive writing, and also keeps the numbers organized.

Examples:

①

$$\begin{cases} x + 2y + 3z = 12 \\ x + 3y + 5z = 21 \\ 5x + 6y + 4z = 12 \end{cases}$$ is encoded as: $$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 1 & 3 & 5 & 21 \\ 5 & 6 & 4 & 12 \end{array}\right)$$

②

$$\begin{cases} x + z = 3 \\ x+y = 2 \\ -y + z = 4 \end{cases}$$ is encoded as: $$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 1 & 1 & 0 & 2 \\ 0 & -1 & 1 & 4 \end{array}\right)$$

③

$$\begin{cases} -y + z = 1 \\ x + z = 3 \\ x + y = 2 \end{cases}$$ is encoded as $$\left(\begin{array}{ccc|c} 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 3 \\ 1 & 1 & 0 & 2 \end{array}\right)$$

The elimination method can be carried out efficiently using the augmented matrix alone:

<u>eg</u> the steps from yesterday, working w/ #1:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 1 & 3 & 5 & 21 \\ 5 & 6 & 4 & 12 \end{array}\right) \xrightarrow[R3 -= 5\cdot R1]{R2 -= R1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & 1 & 2 & 9 \\ 0 & -4 & -11 & -48 \end{array}\right)$$

$$\xrightarrow{R3 += 4\cdot R2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 12 \\ 0 & 1 & 2 & 9 \\ 0 & 0 & -3 & -12 \end{array}\right)$$

Translated back to equations, this is

$$\begin{cases} x + 2y + 3z = 12 \\ \quad\ \ y + 2z = 9 \\ \qquad -3z = -12 \end{cases}$$

which can be solved backwards as shown in class:

$$z = 4 \ ; \quad y = 9 - 2z = 1 \ ; \quad x = 12 - 2y - 3z = 12 - 2\cdot 1 - 3\cdot 4 = -2 \quad \Bigg| \quad (x, y, z) = \underline{(-2, 1, 4)}.$$

<u>Row echelon form</u>

A matrix is in row echelon form if:

1) Any row of all 0's is at the bottom.

2) The first nonzero entry of a row is (strictly) to the right of the previous row's first nonzero entry.

Advantage: the linear system is ready to be solved w/ back-substitution.