### Study guide

- (§2.3) Know and understand the formal definitions of linear dependence and linear independence.
- (§2.3) Understand how linear dependence tells you about redundancy in a list of vectors.
- (§2.3) How do you prove that a list of vectors is linearly independent? Be comfortable with the proof template from class about this.
- (§2.3) If you already know that a list of vectors is linearly independent, how do you make use of this fact in a proof?
- (§6.1) Know the defintion of the dot product as a sum of products of numbers.
- (§6.1) Know the geometric formula for dot product (in terms of $\cos\theta$). (You do not need to know the proof)
- (§6.1) How can you use dot products to measure *lengths* and *angles*?

### Textbook problems

- §1.6: 28
- §2.3: 6, 8, 37, 42

  (*Note* about the wording in 2.3.42: This is two problems in one: first prove the claim stated in the second sentence, about $n \times n$ invertible matrices. The last sentence is asking you to show that the claim is *false* if you don't assume that $A$ is invertible; you should write down a *specific* $2 \times 2$ matrix and two linearly independent vectors $\vec{w}_1, \vec{w}_2$ such that $\{A\vec{w}_1, A\vec{w}_2\}$ is linearly dependent.)

- §6,1: 14, 18, 23, 32

### Supplemental problems:

1. A square matrix $A$ is called *orthogonal* if it is invertible and $A^{-1} = A^t$. (Note: read Theorem 15 in §1.6 of the book. We did not explicitly mention all of these facts in class, but a couple of them are useful here.)

   (a) Prove that if $A$ is orthogonal, then $\det A = \pm 1$.

   (b) Prove that if $A$ is an orthogonal $n \times n$ matrix and $\vec{v}$ is any vector in $\mathbb{R}^n$, then $\|A\vec{v}\| = \|\vec{v}\|$.

   (c) Prove that if $A$ is an orthogonal $n \times n$ matrix and $\vec{u}, \vec{v}$ are any two nonzero vectors in $\mathbb{R}^n$, then the angle between $A\vec{u}$ and $A\vec{v}$ is the same as the angle between $\vec{u}$ and $\vec{v}$.

   (d) Prove that the product of two orthogonal matrices of the same size is orthogonal, and that the inverse of an orthogonal matrix is orthogonal.

   *Note:* Orthogonal matrices arise in physics and engineering, because they represent rigid motions (rotations, etc.), which is demonstrated by the properties discussed above.

2. Suppose that $\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$ is a linearly independent set of vectors in $\mathbb{R}^n$, and let $\vec{v}_{n+1}$ be some other vector in $\mathbb{R}^n$. Prove that $\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n, \vec{v}_{n+1}\}$ is linearly dependent *if and only if* $\vec{v}_{n+1}$ is a linear combination of the vectors $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n$.

   *Note:* be very careful in your proof that you never assume that a number is nonzero without justifying why it must be so.