This document includes only part of PSet 9; it includes only problems about topics that will appear on the midterm. The rest of the problems will be added to this document later. The problem set overall will be longer than usual, since it covers two weeks of material. It is not due until one week after the exam, so that you can focus your energy on studying during the week leading up to the exam.

Study guide

- (lab; see also §6.3)) Understand the *Gram-Schmidt procedure* for turning a basis $B = \{\vec{v}_1, \cdots, \vec{v}_n\}$ into an *orthonormal* basis $\{\vec{u}_1, \cdots, \vec{u}_n\}$.
- (§6.2) Know the definition of an *inner product*, and some examples. Also know the terminology *inner product space.*
- (§6.2) How do you define $\|\vec{v}\|$ and $\vec{u} \perp \vec{v}$ in an inner product space (not necessarily $\mathbb{R}^n$)?
- (§6.2) Understand the proof of the "Pythagorean theorem for inner product spaces."
- (§6.3) Know the definition of the projection $\text{proj}_{\vec{v}}\vec{u}$ in an inner product space, as well as the interpretation as the "nearest multiple" of $\vec{v}$ to $\vec{u}$.

Textbook problems

- §6.2: 2, 4, 10, 12, 29(a,b,c)
- §6.3: 10, 18

Supplemental problems:

1. Let V be an inner product space. Prove that if $S = \{\vec{v}_1, \cdots, \vec{v}_n\}$ is a list of vectors, and $\vec{u}$ is another vector such that $\vec{u} \perp \vec{v}_i$ for all i, then $\vec{u}$ is also orthogonal to any linear combination of the elements of S.

2. Suppose that $\{\vec{u}, \vec{v}, \vec{w}\}$ is an orthogonal set in an inner product space V. Prove that for any three scalars $a, b, c \in \mathbb{R}$,

$$\|a\vec{u} + b\vec{v} + c\vec{w}\|^2 = a^2\|\vec{u}\|^2 + b^2\|\vec{v}\|^2 + c^2\|\vec{w}\|^2.$$

   Hint: apply the Pythagorean theorem for inner product spaces twice.