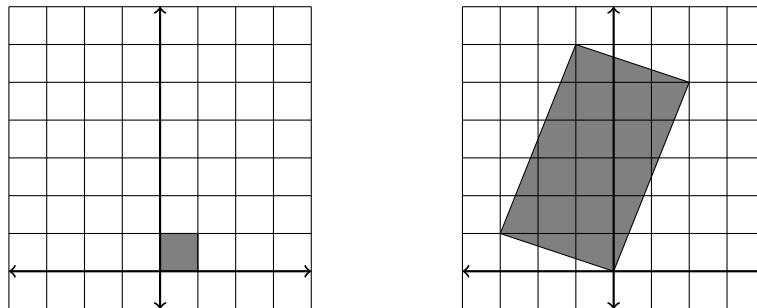


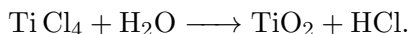
1. [9 points] Solve the following linear system of equations (your answer should describe all possible solutions).

$$\begin{array}{rccccrcr} & & & 3x_3 & +9x_4 & +x_5 & = & 1 \\ -x_1 & +8x_2 & +x_3 & +8x_4 & & & = & 0 \\ x_1 & -8x_2 & +x_3 & -2x_4 & +x_5 & & = & 0 \end{array}$$

2. [9 points] Suppose that A is a 2×2 matrix that transforms the unit square in the plane in the manner shown below.



- Determine A . There are more than one possible answer; you only need to give one.
 - Using the matrix A you found in part (a), determine the area of the parallelogram in the second picture.
3. [9 points] When titanium tetrachloride is sprayed into the air, it reacts with water vapor to form hydrogen chloride and fine particles of titanium dioxide (sometimes used to create smoke screens). The reaction can be expressed in the chemical equation



Write a system of linear equations that could be solved to balance this equation, and express the system using an augmented matrix. **You do not need to solve the system; just the augmented matrix is sufficient.**

4. [9 points] Define \vec{u} and \vec{v} to be the following two vectors. $\vec{u} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} 1 \\ 8 \\ 2 \end{pmatrix}$

- Express $\begin{pmatrix} 2 \\ -11 \\ -5 \end{pmatrix}$ as a linear combination of \vec{u} and \vec{v} .

- Show that $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ is **not** equal to a linear combination of \vec{u} and \vec{v} .

5. [9 points] Suppose that $\vec{u}, \vec{v}, \vec{w}, \vec{b}$ are four vectors in \mathbb{R}^n , such that

- \vec{b} is a linear combination of $\vec{u}, \vec{v}, \vec{w}$, and
- \vec{v} is a linear combination of \vec{u} and \vec{w} .

- (a) Prove that \vec{b} is equal to a linear combination of \vec{u} and \vec{w} alone (not involving \vec{v}).
- (b) Prove that there are infinitely many ways to write \vec{b} as a linear combination of \vec{u} , \vec{v} , and \vec{w} .