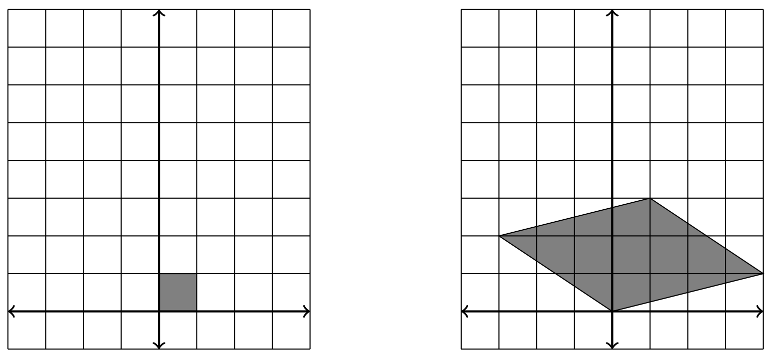
1. [12 points] Find the general solution to the system of equations.

$$\begin{array}{rrrrcr} x_1 & & -x_3 & +4x_4 & = & -3 \\ -x_1 & +x_2 & -2x_3 & -3x_4 & = & 2 \\ x_1 & & -x_3 & +5x_4 & = & -4 \end{array}$$

2. [12 points] Suppose that A is a 2×2 matrix that transforms the unit square in the plane in the manner shown below.



   (a) Determine A. There are more than one possible answer; you only need to give one.

   (b) Using the matrix A you found in part (a), determine the area of the parallelogram in the second picture.

3. [12 points] One of the following two matrices is invertible, and the other is not. Determine which one is invertible, and find its inverse.

$$A = \begin{bmatrix} 2 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -2 & -4 & -2 \\ 0 & 2 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 2 & 4 & 1 \end{bmatrix}$$

4. [12 points] Define $\vec{u}$ and $\vec{v}$ to be the following two vectors.

$$\vec{u} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} 2 \\ 8 \\ 5 \end{pmatrix}$$

One of the following vectors is a linear combination of $\{\vec{u}, \vec{v}\}$, and the other is not. Express one as a linear combination, and show that the other cannnot be.

$$\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

5. [12 points] Suppose that $\vec{u}, \vec{v}, \vec{w}$ are three different solutions to the matrix equation $A\vec{x} = \vec{0}$. Prove that any linear combination of $\{\vec{u}, \vec{v}, \vec{w}\}$ is also a solution to $A\vec{x} = \vec{0}$.