1. [12 points] Suppose that $B = \{\vec{u}, \vec{v}, \vec{w}\}$ is a basis for a vector space V. Prove that $B' = \{\vec{u}, \vec{v}+\vec{w}, \vec{v}-\vec{w}\}$ is also a basis for V.

2. [12 points] Consider the following set of vectors in $\mathbb{R}^5$.

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \right\}.$$

Find a basis for the span of S and determine $\dim \operatorname{span}(S)$.

3. [12 points] Consider the following 3×4 matrix.

$$A = \begin{bmatrix} 1 & -2 & 1 & 5 \\ 0 & 1 & -2 & -5 \\ 1 & 0 & -3 & -5 \end{bmatrix}$$

Let $W \subseteq \mathbb{R}^4$ denote the set
$$W = \left\{ \vec{x} \in \mathbb{R}^4 : \ A\vec{x} = \vec{0} \right\}.$$

(a) Show that W is a subspace of $\mathbb{R}^4$.

(b) Find a basis for W and determine $\dim W$.

4. [12 points] Define three vectors as follows.

$$\vec{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}.$$

For the values $a, b \in \mathbb{R}$ minimizing $\|a\vec{x} + b\vec{1} - \vec{y}\|^2$. In other words, for which a, b is $a\vec{x} + b\vec{1}$ as close as possible to $\vec{y}$?

5. [12 points] The following two sets are both bases of $\mathbb{R}^3$ (you do not need to prove this).

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \quad B' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

Find the change of basis matrix from B to B', i.e. the matrix $[I]_B^{B'}$.