Think of each problem set as a study aid, more so than just an assignment. To help with this, I'll list the main points that you should try to master in a given week at the top of the problem set, as "study guide," together with the relevant textbook section.

Study guide

- (§1.1) Know the three row operations, and understand why they don't change the solutions of a system of equations.
- Be familiar with the notation I like to use in class (which the book does not use): $R1+ = c \cdot R2, R1\cdot = c$ and $R1 \leftrightarrow R2$ (when talking about rows of a matrix), or $E1+ = c \cdot E2$ (etc.) when discussing a list of equations.
- (§1.1) Practice solving linear systems by the elimination method.
- (§1.2) Know how to convert a system of equations to its augmented matrix and vice versa.
- (§1.2) Know the definition of "row echelon form" (REF) and "reduced row echelon form" (RREF).
- Be able to reason about row-reduction when there are some unknowns in the matrix (e.g. Exercises 1.1.43 and 1.2.49).

1. (Textbook §1.1, problems 12,14,16)
   Solve each system using the elimination method. **Clearly state your steps**, e.g. using a notation like $E2+ = c \cdot E1$ as we've used in class.

   a) $\begin{cases} x + 3y + z = 2 \\ -2x + 2y - 4z = -1 \\ -y + 3z = 1 \end{cases}$ b) $\begin{cases} -x + y + 4z = -1 \\ 3x - y + 2z = 2 \\ 2x - 2y - 8z = 2 \end{cases}$ c) $\begin{cases} -2x_1 + x_2 = 2 \\ 3x_1 - x_2 + 2x_3 = 1 \end{cases}$

2. (Textbook 1.1.43)
   Determine the values of k such that the linear system $\begin{cases} 9x + ky = 9 \\ kx + y = -3 \end{cases}$ has

   (a) No solutions;
   (b) Infinitely many solutions;
   (c) A unique solution.

   Reason carefully, and explain your logic. This problem is in part a warm-up for the careful writing needed in proofs.

⋆ **To do, but not to hand in:** do the odd-numbered problems from 9-19 and 21-27 in §1.2, and check your answers in the back of the book. These are short, and will help you check your understanding of the precise details of row echelon form.

3. (Textbook §1.2, problems 20,32,34,36)
   Convert the gives matrix to reduced row echelon form (RREF). **Clearly state your steps.**

   a) $\begin{bmatrix} -3 & 2 \\ 3 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & 1 \\ 1 & -3 & -3 \\ 1 & 2 & -3 \end{bmatrix}$ c) $\begin{bmatrix} -4 & -2 & -1 \\ -2 & -3 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -3 & -4 & -2 \\ -4 & 2 & 1 & -4 \\ -1 & -3 & 1 & -4 \end{bmatrix}$

4. (Textbook §1.2, problems 38, 40, 48)
Convert the linear system to an augmented matrix. Reduce this matrix to RREF, and use this to write the general solution to the system.

a) $\begin{cases} -3x + y = 1 \\ 4x + 2y = 0 \end{cases}$ b) $\begin{cases} 2x - 4z = 1 \\ 4x + 3y - 2z = 0 \\ 2x + 2z = 2 \end{cases}$ c) $\begin{cases} -3x_1 + 2x_2 - x_3 - 2x_4 = 2 \\ x_1 - x_2 - 3x_4 = 3 \\ 4x_1 - 3x_2 + x_3 - x_4 = 1 \end{cases}$

5. (Textbook 1.2.49)
The augmented matrix of a linear system has the following form. Here a, b, c are some unspecified constants.

$$\left[\begin{array}{rrr|c} 1 & 2 & -1 & a \\ 2 & 3 & -2 & b \\ -1 & -1 & 1 & c \end{array}\right]$$

(a) Determine the values of a, b, and c for which the linear system is consistent.
(b) Determine the values of a, b, and c for which the linear system is inconsistent.
(c) When it is consistent, does the linear system have a unique solution or infinitely many solutions?
(d) Give a specific consistent linear system and find one particular solution.

6. (Textbook 1.8.4)
Balance the chemical equation

$$\begin{aligned} &MnS + As_2Cr_{10}O_{35} + H_2SO_4 \longrightarrow \\ &HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O \end{aligned}$$

For this and other application problems from §1.8 on later sets, you are free to use a computer to do the row-reduction; the arithmetic can get a bit dense.

7. (a) Give three examples of linear systems of 2 equations in 4 variables, as follows: one that is inconsistent, one where the general solution with two free variables, and one where the general solution has three free variables.
(b) Explain why it is impossible for a linear system of 2 equations in 4 variables to have a *unique* solution. (*Hint:* think about where the pivots can occur after reducing to echelon form.)

Important notes

- For full credit, you must show or explain your reasoning.
- You are encouraged to work in groups while solving the problems, but all submitted work must be your own work in your own words.