Study guide

- (§3.4) Know the definition of "coordinates of $\vec{v}$ in basis B", and the shorthand notation $[\vec{v}]_B$.
- (§3.4) If S is the standard basis for $\mathbb{R}^n$, then for all $\vec{v} \in \mathbb{R}^n$, $[\vec{v}]_S = \vec{v}$ (the coordinate vector is the same as the vector itself). Make sure you understand why!
- (§3.4) If you are given a vector $\vec{v}$ and a basis B, how do you compute the coordinates $[\vec{v}]_B$?
- (§3.4) Know the definition of the *change of basis matrix* (also called *transition matrices*) $[I]_B^{B'}$ and how to compute them. Know the basic facts about inverses and products of change of basis matrices.

Terminology note: the textbook says "ordered basis" where we've usually just said "basis." Also, the phrase "transition matrix" means the same as "change of basis matrix."

1. (Textbook §3.4, exercises 1–8: find the coordinates of a given vector in terms of a given basis.) You can check the odd-numbered problems in the back of the book.

2. (Textbook §3.4, exercises 13-18: find the transition matrix from one given basis to another.) You can check the odd-numbered problems in the back of the book.

3. (Textbook 3.4.22)
   Let

$$B_1 = \left\{ \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right\}$$

   and

$$B_2 = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix} \right\}$$

   be two ordered bases for $\mathbb{R}^2$.

   (a) Find $[I]_{B_1}^{B_2}$

   (b) Find $[I]_{B_2}^{B_1}$

   (c) Show that $\left([I]_{B_1}^{B_2}\right)^{-1} = [I]_{B_2}^{B_1}$

   The remaining problems were added after the second midterm, and concern material not covered on it.

4. (Textbook §6.2, # 2,4,10)
   (Checking whether or not something is an inner product)

5. (Textbook §6.2, # 29 parts (a),(b), (c))
   (Some computations in $\mathcal{C}[0,1]$)

6. Prove that if $\vec{u}, \vec{v}$ are orthogonal vectors in an inner product space, then $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2+\|\vec{v}\|^2$. (This is the "Pythagorean theorem for inner product spaces.")

7. (Approximation of functions by polynomials) Consider the function $f(x) = \sin(2\pi x)$ on the interval $[0, 1]$. This function can be regarded as a vector in the vector space $\mathcal{C}[0, 1]$ (this is our notation from class; the book uses the notation $\mathcal{C}^{(0)}([0, 1])$).

   (a) Find the degree-3 polynomial $p(x)$ which approximates $f(x)$ as well as possible, in the sense of minimizing the "total squared error"

   $$\int_0^1 (p(x) - f(x))^2 \, dx.$$

   Note that you may view $p(x)$ as a linear combination of $\{1, x, x^2, x^3\}$, which can play the role of vectors $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$ in our discussion of least-squares.

   (b) Make a plot of both the function $f(x)$ and the polynomial $p(x)$ on the interval $[0, 1]$ to verify that $p(x)$ provides a good approximation.

   Remark: Although it is not necessary for this problem, in applications one often finds these polynomial approximations by first using something called the *Gram-Schmidt process* to replace $\{1, x, x^2, x^3\}$ by an *orthogonal* (or orthonormal) set.