Study guide

- (§1.3) Know how to add, multiply, and transpose matrices. Be aware of any restrictions on the dimensions of the matrices involved. Try odd-numbered problems from 1 to 23 in §1.3 to review these (check your answers in the back).
- (§1.3) The formula for matrix multiplication seems very strange at first. Make sure you understand why it is defined the way it is. It helps to think about some examples.
- (§1.8) Understand how to convert a flow/traffic/circuit problem to a linear system of equations.
- (To be discussed in-class) A 2×2 matrix encodes a (linear) tranformation of the plane. Given such a matrix A, understand how to transform individual points or simple picture (e.g. the unit square). Similarly, understand how to find the matrix A given a picture of its effect.

1. (Textbook §1.3, problems 9 and 10)
   Let $A = \begin{bmatrix} 2 & -3 & -3 \\ -3 & -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \\ 3 & 0 \end{bmatrix}$.
   a) Find AB (you can check your answer in the back of the book.)
   b) Find BA.

   Note For now at least, practice doing the matrix algebra in this and the following problems by hand; working examples will build your intuition. It is fine to check your work with software.

2. (Textbook 1.3.12)
   Let $A = \begin{bmatrix} -2 & -2 & -1 \\ -3 & 2 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & -2 \\ -2 & -2 & 3 \\ -3 & 1 & -3 \end{bmatrix}$. Find AB, or explain why it is not defined.

3. (Textbook 1.3.16)
   Find $(A + 2B)(3C)$, or explain why it is not defined, where
   $$A = \begin{bmatrix} -2 & -3 \\ 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix}.$$

4. (Textbook 1.3.22)
   Find $C(A^t + B^t)$, or explain why it is not defined, where
   $$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 0 & -2 \end{bmatrix}, B = \begin{bmatrix} -3 & 1 & 1 \\ -3 & -3 & -2 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 \\ -1 & -3 \end{bmatrix}.$$

5. (Textbook §1.3.26)
   Let $A = \begin{bmatrix} 0 & 2 \\ 0 & 5 \end{bmatrix}$. Find a 2×2 matrix B that is not the zero matrix, such that AB is the zero matrix.

6. (Textbook §1.3.28)
Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$. Find all matrices of the form $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $AM = MA$.

> **Hint** Write a linear system of equations in variables a, b, c, d to describe the situation.

♣ 7. (Textbook 1.3.37)
Suppose that A is an $n \times n$ matrix. Show that if for each vector $\mathbf{x}$ in $\mathbb{R}^n$, $A\mathbf{x} = \mathbf{0}$, then A is the zero matrix.

> **Note** Above and in the future, the symbol ♣ means that this is a proof problem. Be sure to write in complete sentences, using correct mathematical grammar. The quality of your writing and logic is part of the grading of the problem.

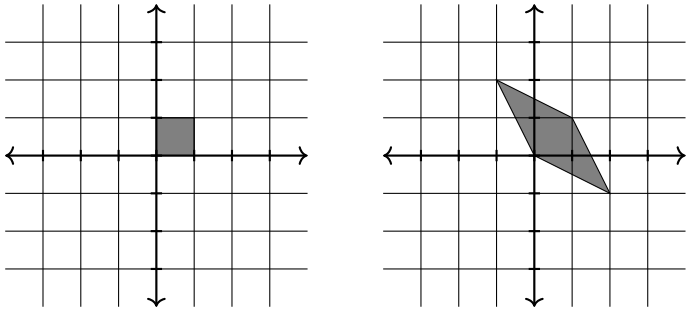
♣ 8. (Textbook 1.3.38)
For each positive integer n, let

$$A_n = \begin{bmatrix} 1-n & -n \\ n & 1+n \end{bmatrix}$$

Show that $A_n A_m = A_{n+m}$.

9. Find a 2×2 matrix A such that $A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$.

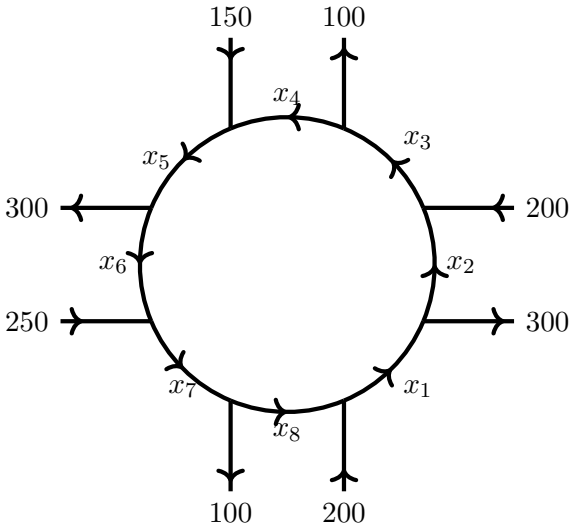
10. A 2×2 matrix A transforms the unit square in the plane in the manner shown below. Determine the matrix A (there is more than one possible answer; you only need to give one).



11. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Draw a pair of pictures like in the problem above to illustrate the way that the matrix A transforms the unit square.

12. (Textbook 1.8.8)

Find the traffic flow pattern for the network in the figure. Flow rates are in cars per half-hour. What is the smallest possible value for x_8, given that all the values $x_1, \cdots, x_8$ must be nonnegative? **You are allowed, and will probably want to, use a computer for the row operations.**

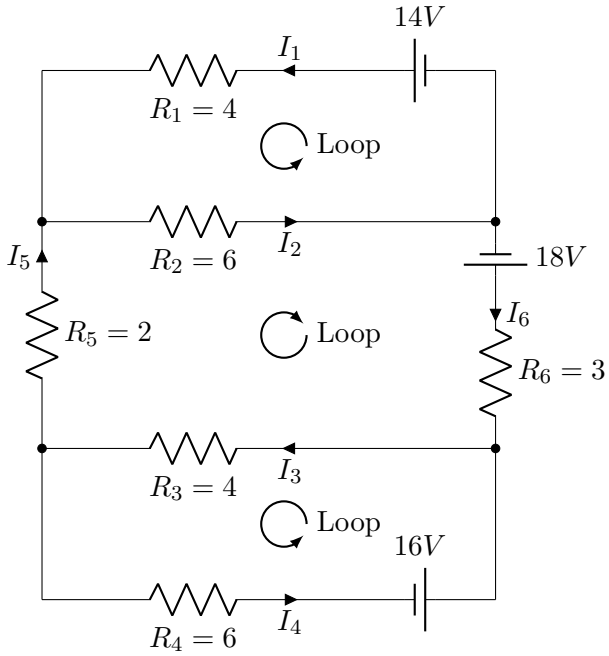


13. (Textbook 1.8.20)
Consider the circuit diagram below (this matches the text, except for one correction: the textbook has the 16 V battery facing the wrong way). Refer to the paragraph before exercise 19 for a brief sumamary of Kirchhoff's laws (we will also discuss them in class).

(a) Apply Kirchhoff's first law to the four junctions to write four equations involving currents.

(b) Apply Kirchhoff's second law to the three loops to write three linear equations.

(c) Solve the system of equations from parts (a) and (b) to find the currents I_1, I_2, I_3, I_4, I_5, and I_6. **You are allowed, and will probably want to, use a computer for the row operations.**



Note You should try problem 19 and check your answer in the back of the book to ensure that you understand the notation and circuit laws.