Study guide

- (§1.5) Understand how to convert a linear system of equations to a matrix equation ($A\vec{x} = \vec{b}$) and vice versa.
- (§1.4) Know the definitions of the identity matrix I_n (or I, when n is clear from context) and the inverse matrix A^{-1} (when it exists).
- (§1.4) Remember: only square matrices (same number of rows as number of columns) can possibly be invertible.
- (§1.4) Be able to quickly invert 2×2 matrices, e.g. using the formula mentioned in class.
- (§1.4) Be able to invert matrices of any size, and to tell when the inverse doesn't exist, by row-reducing a large matrix.
- (§1.5) Know how to use the inverse matrix (if it exists) to solve a matrix equation.

Note For most of this problem set, you'll want to use some facts about inverse matrices to be discussed on Monday 2/17. So while I am posting it now, you probably will want to wait until next week to begin working on it, and it again won't be due until Friday. Alternatively, you can read §1.4 first.

1. (Textbook §1.4, problems 2,6,8,11)
   For each matrix A, find A^{-1} or indicate that it does not exist. when A^{-1} does exist, check your answer by verifying that $AA^{-1} = I$.

   a) $\begin{bmatrix} -3 & 1 \\ 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & 1 \\ -1 & 0 & 0 \\ 2 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & 3 \\ 0 & -1 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -3 & -2 & -3 & 0 \\ 0 & 1 & 3 & 3 \end{bmatrix}$

2. (Textbook 1.4.22)
   Determine those values of λ for which the matrix $\begin{bmatrix} 2 & \lambda & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ is *not* invertible.

3. (Textbook 1.5.14)
   Given that $A^{-1} = \begin{bmatrix} -4 & 3 & -4 \\ 2 & 2 & 0 \\ 1 & 2 & 4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$, solve the linear system $A\vec{x} = \vec{b}$.

4. (Textbook 1.5.18)
   Solve the linear system $\begin{cases} 2x - 4y = 4 \\ -2x + 3y = 3 \end{cases}$ by writing it as a matrix equation $A\vec{x} = \vec{b}$ and inverting A.

♣ 5. (Textbook 1.5.30)
   Suppose that $\vec{u}$ is a solution to $A\vec{x} = \vec{b}$ and that $\vec{v}$ is a solution to $A\vec{x} = \vec{0}$. Show that $\vec{u} + \vec{v}$ is a solution to $A\vec{x} = \vec{b}$.

♣ 6. (a) Suppose that A is an invertible $n \times n$ matrix. Prove that any matrix equation $A\vec{x} = \vec{b}$ (where $\vec{b}$ is known and $\vec{x}$ is unknown) has a *unique* solution.

(b) Suppose that A is a matrix with more columns than rows (i.e. A is $m \times n$, where $m < n$). Prove that any matrix equation $A\vec{x} = \vec{b}$ is either inconsistent or has infinitely many solutions. (*Hint:* after A is reduced to RREF, how many pivots can there be? What does this say about the number of free variables?)

♣ 7. Suppose that A is an invertible matrix. Prove that the transpose A^t is also invertible, and that its inverse is given by $(A^{-1})^t$.

Hint You may wish to make use of some algebraic facts about transposition, summarized in Theorem 6 on page 36 of the textbook.

Note In symbols, the problem above proves $(A^t)^{-1} = (A^{-1})^t$. It is common to use the shorthand A^{-t} to denote this matrix.