

**Study guide**

- (§2.1 and 2.2) Know the definition of “linear combination,” and visual interpretation.
- (§2.2) How can you tell whether a particular vector is a linear combination of some other vectors? How can you describe the set of all vectors that can be written as a linear combination of a specific set of vectors?
- (§2.2) Understand how to interpret a matrix equation  $A\vec{x} = \vec{b}$  in terms of linear combinations of the columns of  $A$ .

- ♣ 1. (a) Let  $A$  be an  $n \times n$  matrix. Suppose that in every row of  $A$ , the entries of that row sum to 0. Prove that  $A$  is not invertible.

**Hint** Use a theorem that we proved in class, giving a criterion for a matrix to be non-invertible.

- (b) Suppose instead that *in every column* of  $A$ , the entries in that column sum to 0. Deduce from part (a) and a problem from the previous assignment that  $A$  is not invertible.

**Note** To clarify the wording: part (a) concerns matrices like  $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 1 & -4 & 3 \end{pmatrix}$  (note that in each of the three rows, the numbers sum to 0), while part (b) concerns matrices like  $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & -4 \\ -3 & -1 & 3 \end{pmatrix}$ .

- ♣ 2. Suppose that  $A$  is a square matrix.

- (a) Prove that if  $A$  has a column consisting entirely of 0s, then  $\det A = 0$ .
- (b) Prove that if two rows of  $A$  are identical, then  $\det A = 0$ .

**Note** We proved a theorem in class very similar to this. For practice, try writing out an argument rather than simply citing that theorem.

- ♣ 3. Suppose that  $A$  is an  $n \times n$  matrix and  $c$  is a constant. Prove that

$$\det(c \cdot A) = c^n \det A.$$

♣ 4. (Textbook 1.6.52)

An  $n \times n$  matrix is skew-symmetric provided  $A^t = -A$ . Show that if  $A$  is skew-symmetric and  $n$  is an odd positive integer, then  $A$  is not invertible.

**Hint** You can use the fact that  $\det A = \det A^t$ . There is also another homework problem that is useful to cite here.

5. (Textbook 2.1.18)

6. (Textbook 2.1.30)

7. (Textbook 2.2.2)

8. (Textbook 2.2.12)

9. (Textbook 2.2.32)

10. (Textbook 2.2.33)

11. (Textbook 2.2.38)