Study guide

- (§2.3) Know and understand the formal definitions of linear dependence and linear independence.
- (§2.3) Understand how linear dependence tells you about redundency in a list of vectors.
- (§2.3) How do you prove that a list of vectors is linearly independent? Be comfortable with the proof template from class about this.
- (§2.3) If you already know that a list of vectors is linearly independent, how do you make use of this fact in a proof?
- (§6.1) Know the defintion of the dot product as a sum of products of numbers.
- (§6.1) Know the geometric formula for dot product (in terms of $\cos\theta$). (You do not need to know the proof)
- (§6.1) How can you use dot products to measure *lengths* and *angles*?

1. (Textbook §2.3, problems 2-10 even)
   (You may also want to work through 1-9 odd and check your answer in the back to practice)
   Determine whether or not the given set of vectors is linearly independent.

   a) $\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix} \right\}$ b) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ c) $\left\{ \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix} \right\}$

   d) $\left\{ \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \right\}$ e) $\left\{ \begin{bmatrix} -2 \\ -4 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -12 \\ 2 \\ 6 \end{bmatrix} \right\}$

♣ 2. (Textbook 2.3.35) Prove that two vectors $\vec{u}, \vec{v}$ in $\mathbb{R}^n$ are linearly dependent if and only if one is a scalar mutliple of the other.

♣ 3. (Textbook 2.3.36) Suppose that $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent and vectors $\vec{w}_1, \vec{w}_2, \vec{w}_3$ are defined by

$$\vec{w}_1 = \vec{v}_1 + \vec{v}_2 + \vec{v}_3, \quad \vec{w}_2 = \vec{v}_2 + \vec{v}_3, \text{ and } \quad \vec{w}_3 = \vec{v}_3.$$

Show that $T = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is linearly independent.

♣ 4. (Textbook 2.3.42, slightly reworded) Let $\vec{v}_1, \dots, \vec{v}_k$ be linearly independent vectors in $\mathbb{R}^n$, and suppose A is an invertible $n \times n$ matrix. Define vectors $\vec{w}_i = A\vec{v}_i$, for $i = 1, \dots, k$.

   (a) Show that the vectors $\vec{w}_1, \dots, \vec{w}_k$ are linearly independent.

   (b) Give a specific example, using a 2×2 matrix, to show that if we don't assume that A is invertible, then part (a) is not necessarily true.

5. (Textbook 6.1.1-4)
   Let $\vec{u} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$. Compute:

   a) $\vec{u} \cdot \vec{v}$ b) $\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}$ c) $\vec{u} \cdot (\vec{v} + 2\vec{w})$ d) $\frac{\vec{u}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w}$

6. (Textbook 6.1.11-16)

   Let $\vec{u} = \begin{bmatrix} -3 \\ -2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix}$.

   a) Find $\|\vec{u}\|$.
   b) Find the distance between $\vec{u}$ and $\vec{v}$.
   c) Find a length-one vector in the direction of $\vec{u}$.
   d) Find the cosine of the angle between the two vectors.
   e) Find a vector in the opposite direction of $\vec{v}$ with length 3.
   f) Find a vector $\vec{w}$ that is orthogonal to both $\vec{u}$ and $\vec{v}$.