Study guide

- (§3.3) Know the definition of a basis. Understand why both parts (spanning, and linear independence) are included.
- (§3.3) Be able to determine whether a given subset of a specific vector space is a basis.
- (§3.3) Be able to find the basis of a subspace defined by several free variables (e.g. problem 3.3.20).
- (§3.3) Be able to find a basis for a span of a set of vectors in $\mathbb{R}^n$ (e.g. problem 3.3.26).
- (§3.3) Know the definition of *dimension.* Make sure you understand the definition, and why it captures the intuitive idea of "degrees of freedom."
- (§3.3) Be familiar with the "standard bases" for $\mathbb{R}^n, \mathcal{P}_d$, and $M_{2\times 2}$.

1. (Textbook 3.3.12)
   Show that the set $S = \left\{x^2+1, x+2, -x^2+x\right\}$ is a basis for the vector space $\mathcal{P}_2$.

2. (Textbook 3.3.20)
   The set $S = \left\{ \begin{bmatrix} a & a+d \\ a+d & d \end{bmatrix} \middle| \; a, d \in \mathbb{R} \right\}$ is a subspace of $M_{2\times 2}$. Find a basis for S, and state its dimension.

3. (Textbook §3.3, problems 25-30)
   (You can check the odd-numbered problems at the back of the book)

4. (Textbook 3.3.38)
   Show that if $S = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is a basis for the vector space V and c is a nonzero scalar, then $S' = \{c\vec{v}_1, c\vec{v}_2, \ldots, c\vec{v}_n\}$ is also a basis for V.

5. (Textbook 3.4.40)
   Find a basis for the subspace $S = \left\{\mathbf{x} \in \mathbb{R}^4 \mid A\mathbf{x} = \mathbf{0}\right\}$ of $\mathbb{R}^4$, where
   $$A = \begin{bmatrix} 3 & 3 & 1 & 3 \\ -1 & 0 & -1 & -1 \\ 2 & 0 & 2 & 1 \end{bmatrix}.$$

6. Suppose that A is an $n \times n$ matrix. Let $W \subseteq M_{n\times n}$ denote the set of matrices B such that $AB = BA$ (that is, A and B commute). You showed on the previous problem set that W is a subspace of $M_{2\times 2}$. Prove that $\dim W \geq 2$.

7. Suppose that $B = \{\vec{u}, \vec{v}\}$ is a basis for a vector space V. Prove that $\{3\vec{u}+2\vec{v}, \vec{u}+\vec{v}\}$ is also a basis for V