Reading HHM §2.2

1. Use algebraic methods to prove the cancellation identity: If n and k are nonnegative integers and m is an integer with $m \le n$ then
$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}.$$

2. Suppose that a museum curator with a collection of n paintings by Jackson Pollack needs to select k of them for display and needs to pick m of these to put in a particularly prominent part of the display. Show how to count the number of possible combinations in two ways so that the cancellation identity appears.

3. Prove the parallel summation identity (hockeystick theorem): If m and n are nonnegative integers then
$$\sum_{k=0}^{n} \binom{m+k}{k} = \binom{m+n+1}{n}.$$

4. Compute the value of the following sums. Your answer should be an expression involving one or two binomial coefficients.
   (a) $\displaystyle\sum_{k=100}^{201} \sum_{j=100}^{k} \binom{201}{k+1}\binom{j}{100}$.
   (b) $\displaystyle\sum_{k} \binom{n}{k}^2$ for a nonnegative integer n.

5. Let n be a nonnegative integer. Suppose $f(x)$ and $g(x)$ are functions defined for all real numbers x and that both functions are n times differentiable. Let $f^{(k)}(x)$ denote the kth derivative of $f(x)$; so $f^{(0)}(x) = f(x)$, $f^{(1)}(x) = f'(x)$ and $f^{(2)}(x) = f''(x)$. Let $h(x) = f(x)g(x)$. Show that
$$h^{(n)}(x) = \sum_{k} \binom{n}{k} f^{(k)}(x) g^{(n-k)}(x).$$

6. The state of Florida administers several lottery games. In Florida Lotto a player picks a set of six numbers between 1 and 53. In Fantasy 5 a player chooses a set of five numbers between 1 and 36. In which game is a player more likely to match at least two numbers against the ones drawn?