Reading HHM 2.3

Note We are moving back to **Wednesday** due dates with this assignment. The assignment is a little shorter than normal since you only have five days to complete it.

1. Call a sequence of digits $(d_1, d_2, \cdots, d_\ell)$ *decreasing* if $d_1 > d_2 > \cdots > d_\ell$, and *nonincreasing* if $d_1 \geq d_2 \geq \cdots \geq d_\ell$. Find and prove a formula for the number of decreasing sequences of digits of length ℓ, and a formula for the number of nonincreasing sequences of digits of length ℓ. Here a *digit* means one of the numbers $d \in \{0, 1, 2, \cdots, 9\}$.

2. Prove the addition identity for multinomial coefficients (2.20) by using the expansion identity (2.18) (the numbers refer to formula numbers in the textbook).

3. For nonnegative integers a, b, and c, let $P(a, b, c)$ denote the number of paths in three-dimensional space that begin at the origin, end at (a, b, c), and consist entirely of steps of unit length each of which is parallel to a coordinate axis. Prove that $P(a, b, c) = \binom{a+b+c}{a,b,c}$.

4. Prove the following identities for sums of multinomial coefficients. Assume m and n are positive integers.

   (a) $\displaystyle\sum_{k_1+\ldots+k_m=n} \binom{n}{k_1, \ldots, k_m} = m^n.$

   (b) $\displaystyle\sum_{k_1+\ldots+k_m=n} \binom{n}{k_1, \ldots, k_m} (-1)^{k_2+k_4+\ldots+k_{2\ell}} = \begin{cases} 0, & \text{if } m = 2\ell \\ 1, & \text{if } m = 2\ell+1 \end{cases}.$

5. Prove that if n is a nonnegative integer and k is an integer, then

$$\sum_j \binom{n}{j, k, n-j-k} = 2^{n-k} \binom{n}{k}.$$