

1. Suppose 50 socks lie in a drawer. Each one is either white or black, ankle-high or knee-high, and either has a hole or doesn't. 22 socks are white, four of these have a hole and one of these four is knee-high. Ten white socks are knee-high, ten black socks are knee-high, and five knee-high socks have a hole. Exactly three ankle-high socks have a hole.
  - (a) Use Theorem 2.6 to determine the number of black ankle-high socks with no holes.
  - (b) Draw a Venn diagram that shows the number of socks with each combination of characteristics.
2. On a busy evening, a number of guests visit a gourmet restaurant and everyone orders something. 140 guests order a beverage, 190 order an entree, 100 order an appetizer, 90 order a dessert, 65 order a beverage and an appetizer, 125 order a beverage and an entree, 60 order a beverage and a dessert, 85 order an entree and an appetizer, 75 order an entree and a dessert, 60 order an appetizer and a dessert, 40 order a beverage, appetizer, and dessert, 55 order a beverage, entree, and dessert, 45 order an appetizer, entree, and dessert, 35 order a beverage, entree, and appetizer, and ten order all four types of items. Use Theorem 2.6 to determine the number of guests who visited the restaurant that evening.
3. Suppose that a set of  $n$  cards are chosen from a standard deck of cards. How many ways are there to choose these  $n$  cards such that there is at least one card of each suit? You may answer in terms of binomial coefficients.
4. Is the probability that a permutation of  $n$  objects is a derangement substantially different for  $n = 12$  and  $n = 120$ ? Quantify your answer.
5. (Deranged twins.) Suppose  $n + 2$  people are seated behind a long table facing an audience to staff a panel discussion. Two of the people are identical twins wearing identical clothing. At intermission, the panelists decide to rearrange themselves so that it will be apparent to the audience that everyone has moved to a different seat when the panel reconvenes. Each twin can therefore take neither her own former place nor her twin's. Let  $T_n$  denote the number of different ways to derange the panel in this way.
  - (a) Compute  $T_0, T_1, T_2$ , and  $T_3$ .
  - (b) Compute  $T_4$ .
  - (c) Determine a formula for  $T_n$  and check that your formula produces  $T_{10} = 72, 755, 370$ .
  - (d) Compute the value of  $\lim_{n \rightarrow \infty} \frac{T_n}{(n+2)!}$ .