1. HHM §2.6.6 (p. 189-190), exercise 4.
2. HHM §2.6.6 (p. 189-190) exercise 6.
3. The nth Motzkin number $M_n (n \in \mathbb{N})$ is the number of paths from $(0, 0)$ to $(n, 0)$ in which
   - one can move either one unit to the right from (x, y) to $(x + 1, y)$, diagonally up to the right from (x, y) to $(x + 1, y + 1)$, or diagonally down to the right from (x, y) to $(x + 1, y - 1)$, and
   - the path can not go below the x-axis.

   By convention, we define $M_0 = 1$.

   (a) Draw the allowed Motzkin paths for $n = 1, 2, 3, 4$, and verify that $M_1 = 1, M_2 = 2, M_3 = 4, M_4 = 9$.

   (b) It can be shown that for integers $n \geq 2$, the Motzkin numbers satisfy the recursion

   $$M_n = M_{n-1} + \sum_{k=0}^{n-2} M_k M_{n-k-2}.$$

   Use this to show that the generating function

   $$M(x) = \sum_{n=0}^{\infty} M_n x^n$$

   for the Motzkin numbers has the following closed form:

   $$M(x) = \frac{1 - x - \sqrt{1 - 2x - 3x^2}}{2x^2}$$

4. How many different necklaces having five beads can be formed using three different kinds of beads if we discount:

   (a) Both flips and rotations? (It may be helpful to consult the description of the dihedral group D_5 on p. 194 of the textbook)

   (b) Rotations only?

   (c) Just one flip?

5. The dihedral group D_6 encodes all rotations and reflections of a hexagon.

   (a) List the 12 elements of the dihedral group D_6 in cycle notation.

   (b) How many necklaces of 6 beads can be made from beads of n different colors? Answer as a polynomial in n. Note that a necklace is "the same" if it is rotated or flipped over (reflected). (Use Burnside's lemma)

   (c) How many disctinct necklaces of 6 beads can be made from 2 red beads, 2 blue beads, and 2 green beads? (Use Burnside's lemma; you will find that you'll have to compute C_π in a different way than in part (b). It is okay to take shortcuts in your computation, e.g. by saing things like "the following several permutations all behave the same way.")