Amherst College
Department of Mathematics and Statistics

Math 350-01 | Midterm 2 Practice | Fall 2018

Name:____

Read This First!

- Keep cell phones off and out of sight.
- Do not talk during the exam.
- You are allowed one page of notes, front and back. No other books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted.
- Please read each question carefully. Show **ALL** work clearly in the space provided. There is an extra page at the back for additional scratchwork.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	Total
Points:	8	8	8	8	8	40
Score:						

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1. [8 points] Let G, H be two groups. Prove that $G \times H$ is isomorphic to $H \times G$.

2. [8 points] Prove that if G is a cyclic group, then there exists a surjective group homomorphism $\phi : \mathbb{Z} \to G$.

3. [8 points]] Let R be a ring, and $a \in R$ an element.

   (a) Prove that if a is not a zero-divisor, and $b, c \in R$ satisfiy $ab = ac$, then $b = c$.

   (b) Prove that if a *is* a zero-divisor, then there exist two elements $b, c \in R$ with $b \neq c$ but $ab = ac$.

4. [8 points] Suppose that G is an abelian group, and let H be the set of all elements of G with finite order.

   (a) Prove that H is a normal subgroup of G.

   (b) Prove that all elements of G/H besides the identity have infinite order.

5. [8 points] Let $\phi : R \to S$ be a ring homomorphism.

   (a) Define $\ker \phi$.

   (b) Prove that $\ker \phi$ is an ideal of R.

   (c) Prove that if S is a field, R is a commutative ring with unity, and ϕ is surjective, then $\ker \phi$ is a *maximal* ideal of R.