NOTE: To give you more flexibility as you split your time between the problem set, the project, and studying for the exam: the score that will count in your grade will be 1.25 times your original score, up to a maximum of 100% of the original points. Therefore any score above 80% of the possible points will be counted as 100%.

- **Read:** §18.
- **Suggestion:** Work (or think about) the following problems. Problems marked with a $*$ have answers given at the back of the book.
  - §17 : $1*, 6*$
  - §18 : $1*, 7*$

1. Consider two ideals $I = m\mathbb{Z}$ and $J = n\mathbb{Z}$ in the ring $\mathbb{Z}$, where m and n are positive integers.
   (a) Define $I + J = \{x + y : \ x \in I, y \in J\}$ (as in class). Prove that $I + J = (m, n) \cdot \mathbb{Z}$ (here (m, n) is the greatest common divisor of m and n).
   (b) Prove that $I \cap J = L \cdot \mathbb{Z}$, where L is the least common multiple of m and n.
2. Let R be a ring, and suppose I, J are two ideals of R.
   (a) Prove that $I \cap J$ is always an ideal.
   (b) If I and J are both prime ideals, does it follow that $I \cap J$ must be a prime ideal? Either prove that it must be prime, or provide a counterexample.
3. A *principal ideal domain* (PID) is an integral domain in which every ideal I in R is equal to $R \cdot a$ for some $a \in R$.

   Prove that if R is a PID, then every nontrivial proper prime ideal is maximal.
4. Prove Theorem 18.4 from Saracino.
5. Prove Theorem 18.6 (the second isomorphism theorem) from Saracino.