General comment: Reading the textbook will be an important part of the course. The book's style is very conversational, and is meant to help you see not just the content, but how to think about and learn the subject. Sometimes, you will need to read information or definitions in the book before attempting certain problems. Please remember that you can always ask me, Allison Tanguay, or Dana Frishman for help or suggestions on how to use the book effectively!

Suggested reading for this assignment: *Algebra in Action* sections 1.1 and 1.2. You may also want to read ahead to sections 1.3 and 1.4, which we will discuss in class next week.

Problems on sets and induction The following three problems are meant to review some mechanics of proof-writing, especially proof by induction. Please ask one of the course staff for help if you are new or rusty on induction, or if you have other questions!

1. Let A, B, C be sets. Prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

(This is sometimes phrased: "$\cup$ distributes over $\cap$.")

2. Suppose that x is a nonzero real number such that $x + \frac{1}{x}$ is an integer. Prove by induction that $x^n + \frac{1}{x^n} \in \mathbb{Z}$ for all $n \in \mathbb{Z}^+$.

3. Define the *Fibonacci sequence* f_1, f_2, f_3 as follows:

$$f_1 = f_2 = 1,\ f_3 = 2,\ f_4 = 3,\ f_5 = 5,\ f_6 = 8, \cdots$$

and in general

$$f_n = f_{n-1} + f_{n-2} \text{ for all } n \geq 3.$$

Prove that for all $n \geq 1$, $\sum_{k=1}^{n} f_k^2 = f_n f_{n+1}$. (*Hint:* use induction.)

Textbook problems. These problems are from the textbook, but I have printed the problem numbers for convenience. Some problems (but not all) have hints or solutions in the appendices; you should first try to solve the problem, and then check the back of the book when you are stuck (that is when you are in the best position for new ideas to stick!).

4. (**1.1.1**) Complete the multiplication table for D_8. Find at least one interesting pattern in the table.

For the following problem, read the definition of "the center" on page 9 of the book.

5. (**1.1.6**) Let D_6 denote the set of symmetries of an equilateral triangle. Find the multiplication table for D_6. What is the center of D_6?

The remaining problems concern §1.2 of the book, which we will begin on Friday. You may want to wait until Friday to work on these, or read ahead in the book to learn the definitions.

6. (**1.2.1**) Let $\Omega = \mathbb{Z}$ be the set of integers. Define $f : \Omega \to \Omega$ by $f(x) = x + 5$. Is $f \in \text{Perm}(\Omega)$? If so, what is its inverse? If n is a positive integer, then what is $f^n(x)$? What if instead of $\mathbb{Z}$, we had $\Omega = \mathbb{Z}^{\geq 0}$, the set of non-negative integers?

7. (**1.2.4**) Let $\sigma = (1\ 3\ 5)(2\ 4)$ and $\tau = (1\ 5)(2\ 3)$ be elements of S_5. Find σ^2, $\sigma\tau$, $\tau\sigma$, and $\tau\sigma^2$.

8. (**1.2.6**) Let $f = (1\ 2\ 3) \in S_3$. Find the maps in the following sequence

$$1_{[3]}, f, f^2, f^3, f^4, f^5, \ldots$$

   Do you see a pattern?

   For the following problem, reading the defintion of "right inverse" on page 19 (above the problem).

9. (**1.2.15**)

   (a) Give an example of a map f that has a right inverse, but not an inverse.

   (b) Show that f has a right inverse if and only if f is onto.

10. (**1.2.20**) Let S be a set with a finite number of elements, and let $f : S \to S$ be a map.

    (a) If f is onto, can f not be 1-1?

    (b) If f is 1-1, can f not be onto?

    (c) Do your conclusions remain valid even if S has an infinite number of elements?

Some other good problems to try for additional review and practice (but not to hand in):
1.1.3, 1.1.4, 1.2.2, 1.2.5