

1. **(1.3.4)** Let $\mathbb{Z}/9\mathbb{Z} = \{0, 1, \dots, 8\}$ with addition and multiplication mod 9. Define $f : \mathbb{Z}/9\mathbb{Z} \rightarrow \mathbb{Z}/9\mathbb{Z}$ by $f(a) = 5a$. Does f have an inverse? Is f 1-1? Is f onto? Answer the same questions for the map $g : \mathbb{Z}/9\mathbb{Z} \rightarrow \mathbb{Z}/9\mathbb{Z}$ defined by $g(a) = 3a$.
2. **(1.3.6)** Let $G = \{3, 9, 15, 21\}$, and let the operation on G be multiplication mod 24. Is the operation closed? Is there an identity element? Does every element have an inverse?
3. **(1.3.9)** Find the multiplication table for $(\mathbb{Z}/8\mathbb{Z})^\times$ explicitly.
4. **(1.4.2)** Let $a = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix} \in \text{GL}(2, 3)$. What is the inverse of a ? Is $a \in \text{SL}(2, 3)$?
5. **(2.1.4)** $\text{GL}(n, \mathbb{Z})$ and $\text{SL}(n, \mathbb{Z})$. Let $M_{n \times n}(\mathbb{Z})$ be the set of $n \times n$ matrices with integer entries.
 - (a) Does the set of invertible matrices in $M_{n \times n}(\mathbb{Z})$ form a group?

The set of invertible matrices in $M_{n \times n}(\mathbb{Z})$ whose matrix inverse is also a matrix in $M_{n \times n}(\mathbb{Z})$ is denoted by $\text{GL}(n, \mathbb{Z})$. In addition, the set of matrices in $M_{n \times n}(\mathbb{Z})$ that have determinant 1 is denoted by $\text{SL}(n, \mathbb{Z})$.

 - (b) Let $A \in M_{n \times n}(\mathbb{Z})$. Show that $A \in \text{GL}(n, \mathbb{Z})$ if and only if $\det(A) = \pm 1$.
 - (c) Is $\text{GL}(n, \mathbb{Z})$ a group?
 - (d) Is $\text{SL}(n, \mathbb{Z})$ a group?
6. **(2.1.5)** Let $M_{2 \times 2}(\mathbb{Z}/6\mathbb{Z})$ be the set of 2×2 matrices with entries in $\mathbb{Z}/6\mathbb{Z}$.
 - (a) Can you find a matrix in $M_{2 \times 2}(\mathbb{Z}/6\mathbb{Z})$ whose determinant is non-zero and yet is not invertible?
 - (b) Does the set of invertible matrices in $M_{2 \times 2}(\mathbb{Z}/6\mathbb{Z})$ form a group?

Note: The following problem uses the following terminology: a group is *abelian* if the group operation is commutative.
7. **(2.2.1)** Let G be a group. Prove that $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G if and only if G is abelian.
8. **(2.2.4)**
 - (a) If G is a finite group of even order, show that there must be an element $a \neq e$, such that $a^{-1} = a$.
 - (b) Give an example to show that the conclusion of part 8a does not hold for groups of odd order.

Some other good problems to try for additional practice (but not to hand in):

1.3.1, 1.3.2, 1.4.4, 1.4.6, 2.1.1, 2.1.3, 2.1.6, 2.2.3