

Note: due to the midterm exam on Friday 10/11:

- This assignment is shorter than usual.
- You can skip 20% of the set and still earn full points. More precisely: when computing grades, I will reduce all scores above 80% down to 80%, and then divide by 0.8. Of course, you will still receive feedback and scores for all problems you submit.

These policies are meant to give you more flexibility in allocating your time this week.

- (Not to hand in; but highly recommended) While you read the textbook this week, carefully study the textbook's proof that no permutation is both even and odd (Theorem 3.10, p. 80), which is very different from the argument I'll show in class. This will give you two points of view on this fact, and more practice reasoning about factorization in S_n .
1. Let G be a cyclic group of order n . Prove that the number of generators of G is equal to $\phi(n)$ (see Definition 1.45 on p. 23 for the definition of $\phi(n)$, or your class notes from 9/11).
 2. **(2.6.24) Groups with only trivial subgroups.** The finite group G has more than one element and no nontrivial subgroups. Prove that G is cyclic of order p , where p is a prime number.
 3. Any transposition of the form $(a, a + 1)$ is called an *adjacent transposition*, or a *simple transposition*. Prove that any transposition (a, b) can be factored into a product of an *odd number* of adjacent transpositions. We will use this in class as a stepping stone to proving that no permutation is both even and odd, so **do not make use of this theorem** in your proof.
 4. **(3.1.14)** Let $n > 1$ be an integer, and let $\tau = (1\ 2)$ and $\sigma = (1\ 2\ \dots\ n)$ be elements of S_n .
 - (a) What is $\sigma\tau\sigma^{-1}$? What about $\sigma^2\tau\sigma^{-2}$?
 - (b) Show that $\langle\tau, \sigma\rangle$ contains every simple transposition (see Problem 3.1.13).
 - (c) Do τ and σ generate S_n ?
 - (d) Let a and b be integers with $1 \leq a < b \leq n$. Let σ' be any n -cycle that sends a to b , and let $\tau' = (a\ b)$. Show that $\langle\sigma', \tau'\rangle = S_n$.
 5. **(3.2.4)** How many elements of order 2 does A_5 have?
 6. **(3.2.10)** Let $n \geq 3$. Prove that A_n can be generated by all the 3-cycles in S_n .

Some other good problems to try for additional practice (but not to hand in): 3.1.1, 3.1.4, 3.1.5, 3.2.2, 3.2.3, 3.2.5, 3.2.8