Instructor: Nathan Pflueger (pronounced "fleeger") **office hours**: TBA
email: npflueger@amherst.edu
office: SMUD 401

Time and location: Mon, Wed, Fri 1:00-1:50 SMUD 205

Course webpage `http://npflueger.github.io/385/`
Please consult this site for most course materials (I rarely use Moodle). No login is required.

How to reach me Come to office hours! No appointment is needed. Besides that, I generally reply to email within 24 hours. However, **I may not read or reply to email on weekends, outside business hours, or on Thursdays**, which is the day I devote primarily to research.

Course outline

Mathematical logic is sometimes called "metamathematics:" our aim is to study theorems and proofs as mathematical objects in their own right. Logic sits at the intersection of mathematics, philosophy, and computer science. Our point of view is that of mathematics: we will construct a formal model for the work of a mathematician, and prove theorems about it as we would about any other mathematical model. There are three main parts to this course, of roughly four weeks each.

Part one: syntax and semantics of first-order logic (§1) Our study of mathematical logic begins by considering how to represent the languages that mathematicians work with. What is a language in this sense, and what features of a language are important to mathematicians? This leads to the syntax of so-called *first-order logic.* We then turn to the question of interpretation, or semantics, of the symbols in a language. How does the language used by a mathematician correspond to the actual concepts they are trying to describe. What does it mean then to say that a statement in a language is true?

Part two: proofs and the completeness theorem (§2-3) Once we have the foundation set up, we can turn to the main questions of the course. What is a proof? If you can prove something, does that mean it is true? If something is true, does that mean you can prove it? The highlight is Gödel's remarkable *completeness theorem.*

Part three: the incompleteness theorems (§4-6) Some of the landmark pieces of work in all of mathematics are Gödel's Incompleteness Theorems (not to be comfused with the less famous but still remarkable completeness theorem – by the end of the course you will understand how theorems with both these names can both be true!). Essentially, these say that, within a suitable logical system, there are statements that must be true which we can not hope to prove (at least within that system). In this part of the course, we will outline the proofs of these theorems, though there will not be sufficient time to present all the technical details.

Prerequisites solid experience with mathematical proofs, at least at the level of Math 220 or 271/2. You should be comfortable working with abstract mathematical definitions, and writing proofs of mathematical state- ments using standard proof techniques. One of the challenges of this

course is that we will be studying notions of logic and proof, but also using logic and proof in that study. You should welcome the confusion that this challenge can entail.

Textbook Leary and Kristiansen, *A Friendly Introduction to Mathematical Logic*, **2nd edition**. Milne Library. **The textbook is available in pdf for free**:

`https://milneopentextbooks.org/a-friendly-introduction-to-mathematical-logic/`

There will also be assigned readings from the book *Gödel's Proof* (revised edition) by Nagel and Newman. This book will be made available for free via Moodle.

Course structure

There will be homework due most Wednesdays at 10pm, one take-home midterm exam, and one take-home final exam.

Grading: Grades are based on the following categories. The exact cutoffs for each letter grade are not set in advance; I calibrate them at the end based the difficulty and score distribution of the exams. There is no set curve, but typically the median grade is around B+.

Participation	5%	
Homework	40%	usually due Wednesdays at 10pm
Midterm exam	20%	Take-home, most likely the week after spring break
Final exam	35%	Take-home, due during finals week

Expectations You should expect to spend at least eight hours studying and working on problem sets outside of class each week. Of that time, I recommend that you spend at least two hours reviewing your notes, the textbook, and previous assignments. Distributing your practice and review throughout the semester will be much more effective than concentrating your review and studying right before exams or due dates. You are expected to attend class every day, arrive on time, and be respectful. You are expected to know about any announcement I make in class or by email.

I encourage you to **stop me to ask questions**. Active participation helps but your brain in the mode that will make new connections and learn well. If you are feeling lost, there is almost certainly someone else feeling the same thing; asking a question may help many of your classmates as well!

Course policies

Dropped assignments To compensate for illness and other emergencies, your **lowest two homework scores will be dropped**. If you cannot make a due date due to an emergency, my advice is to skip the assignment, but study and understand the problems when you have time, and focus on keeping up with the new material in the course. You do not need to apologize or provide any reasons for skipping an assignment or turning it in unfinished; please choose what is best for your time, health, and well-being. Remember that **the primary purpose of the homework is not evaluation, but to help you learn the material and guide your studying**, so you should still work through all problems on any assignment your drop, and ask me about them as needed.

Homework deadlines and late policy Homework will be **due at 10pm**, typically on Wednesdays and Fridays, via Gradescope. To allow for technical difficulties or other last-minute issues, Gradescope will allow you to submit homework after the deadline, however your score will be reduced by 2% **per hour** after the deadline (scaled continuously, e.g. being fifteen minutes late results in a 0.5% deduction). Please try to turn in your work by the due time (I don't want to be responsible for lost sleep!), but don't worry about short delays. **I generally do not grant extensions**, but instead drop two assignments (see above).

Accommodations I strive to make this course welcoming to all students. If you would like to discuss your learning needs with me, please schedule a meeting so that we can work together to support your academic success. Anyone who may require an accommodation based on the impact of a disability should contact me to make arrangements. I rely on Accessibility Services for assistance in verifying the need for accommodations and developing accommodation strategies, so you should contact them at `accessibility@amherst.edu` or 413-542-2337. If you require accommodations on exams, please arrange this with me at least one week in advance.

Intellectual responsibility

- **Homework:** Mathematics is a collaborative subject; open and generous communication is one of its core values. Therefore you are strongly encouraged to work with other students, ask many questions, and learn from as many people as possible. However, you must write up the solution yourself. **All your submitted work must be your work, written in your own words**. Copying solutions from other students, solutions manuals, online databases, or generative AI is plagiarism; such copying will result in a 0 on the assignment and will be reported to Community Standards. You are also expected to **list each person your worked with** on the front of your homework assignment.
- **Take-home exams:** absolutely no collaboration or communication with other students, or use of online forums or generative AI, is allowed on exams. You will not be permitted to discuss the exam problems with anyone apart from me before the exam due date. Any collaboration on exams is considered cheating and is a violation of the Honor Code.

For homework and exams, I reserve the right to give no credit for any work that appears suspicious.

Tips and resources

Come to office hours! I am happy to answer your questions and also talk about the course in general. Even if you don't have specific questions, you can come to review material, listen to other students' questions, or just to chat. There is a desk in my office and several just outside where you are welcome to work, chat, and listen in. Office hours are the best way I have to learn about you and how you're doing in the course and the college, so please visit!

Focus on practice and improvement. Every homework problem, or example and class or the book is an opportunity to practice. Take these opportunities, and make the most of them!

Distribute your practice. Study a bit every day, not just before exams. Treat every homework problem as a chance to practice and study.

Actively seek opportunities to practice. Ask me questions, ask classmates questions, read examples in the book, and try problems that haven't been assigned.