1. Write out a complete (not abbreviated, this time) deduction in our proposition calculus of formula $\alpha \to \neg\neg\alpha$. Note that for this problem, you should write a deduction in full, i.e. each line is a premise, an axiom, or follows from previous lines by modus ponense. **In all other problems on this problem set, it is enough to write an "abbreviated deduction" as we have defined in class.**

2. Recall that, in our formulation of first-order logic, the $\wedge$ symbols is an *abbreviation*: $\alpha \wedge \beta$ is always understood to be *literally the same formula* as $\neg(\neg\alpha \vee \neg\beta)$. Give an abbreviated deduction (in the sense stated in class) showing each of the following. Together these give the main tools for interacting with the $\wedge$ symbol.

   (a) $\vdash \alpha \wedge \beta \to \alpha$

   (b) $\vdash \alpha \wedge \beta \to \beta$

   (c) $\vdash \alpha \to (\beta \to \alpha \wedge \beta)$

3. Give an abbreviated deduction for each of the following *associativity* formulas for $\vee$. Note: if you wish, you may prove one and then use it as a premise in your deduction of the other.

   (a) $\vdash (\alpha \vee \beta) \vee \gamma \to \alpha \vee (\beta \vee \gamma)$

   (b) $\vdash \alpha \vee (\beta \vee \gamma) \to (\alpha \vee \beta) \vee \gamma$

4. We proved one version of "contraposition" in class, namely

$$\alpha \to \neg\beta \vdash \beta \to \neg\alpha.$$

   Prove the following other version of contraposition:

$$\alpha \to \beta \vdash \neg\beta \to \neg\alpha.$$

   You may assume in your argument that we have already demonstrated the two "double negation" rules (DN1) and (DN2) from class.

5. Recall that the Propositional Deduction Theorem (PDT) states that for any set of formulas $\Gamma$ and any two formula $\alpha$ and $\beta$,

$$\Gamma, \alpha \vdash_P \beta \text{ if and only if } \Gamma \vdash_P \alpha \to \beta.$$

   We stated this theorem in class on Friday, but haven't proved it yet. Prove the easier direction of the PDT: if $\Gamma \vdash_P \alpha \to \beta$, then $\Gamma, \alpha \vdash_P \beta$.

6. In the following problems, you may assume that we have already proved the Propositional Deduction Theorem (PDT), as stated above. You are also free to assume that any statements you proved earlier on this problem set (e.g. Problem 2) are true if $\vdash$ is replaced with $\vdash_P$.

   (a) Prove that $\alpha \to (\beta \to \gamma) \vdash_P \alpha \wedge \beta \to \gamma$.

   (b) Prove conversely that $\alpha \wedge \beta \to \gamma \vdash_P \alpha \to \beta \to \gamma$.

   (c) Prove that for any set of formulas $\Gamma$, $\Gamma \vdash \alpha \to (\beta \to \gamma)$ if and only if $\Gamma \vdash \alpha \wedge \beta \to \gamma$.