Reading Stewart §2.1, 3.1-3.2

1. Suppose that L/K is a field extension, and $[L:K]$ is a prime number. Prove that, for *any* $\alpha \in L$ such that $\alpha \notin K$, we have $L = K(\alpha)$.

2. Suppose that $K \subseteq L \subset \mathbb{C}$ are subfields, and that L/K is a finite extension. Let $\alpha \in \mathbb{C}$ be any complex number. Prove that α is algebraic over L if and only if α is algebraic over K.

3. (Textbook 2.4)
   Show that $\partial(f+g)$ can be less than $\max(\partial f, \partial g)$, and indeed that $\partial(f+g)$ can be less than $\min(\partial f, \partial g)$.

4. (Textbook 3.1)
   For the following pairs of polynomials f and g over $\mathbb{Q}$, find the quotient and remainder on dividing g by f.
   (a) $g = t^7 - t^3 + 5, f = t^3 + 7$
   (b) $g = t^2 + 1, f = t^2$
   (c) $g = 4t^3 - 17t^2 + t - 3, f = 2t + 5$
   (d) $g = t^4 - 1, f = t^2 + 1$
   (e) $g = t^4 - 1, f = 3t^2 + 3t$

5. (Textbook 3.2)
   Find gcd's for these pairs of polynomials, and check that your results are common factors of f and g.

6. (Textbook 3.3)
   Express these gcd's in the form $af + bg$.

7. Let $K \subseteq L \subseteq \mathbb{C}$ be subfields, and suppose that $[L:K] = 2$ (L is called a *quadratic extension* of K). Prove that L is obtained from K by adjoining some square root. That is, there exists some $\alpha \in K$ such that $L = K(\sqrt{\alpha})$. (This one is a bit tricky. Don't be shy about asking for a hint if you've worked on it for a bit and are stuck.)

8. (Bonus, for a small amount of extra credit). Did you need to assume that K, L were subfields of $\mathbb{C}$ in the argument above? See if you can state a sufficient hypothesis on a general field K guaranteeing that any quadratic extension of K is given by adjoining a square root.