1. Prove that if L/K is a finite field extension, and $\phi : K \to \Omega$ is a nonzero field homomorphism from K to an algebraically closed field Ω, then there exists an extension of K to L. That is, there exists a field homomorphism $\psi : L \to \Omega$ such that $\psi|_K = \phi$.

   Hint First consider the case where L/K is a simple extension. Then use induction to obtain the general result.

2. Prove the following extension of Problem 7 on PSet 4: if L/K is a finite extension in $\mathbb{C}$, $\alpha \in L$, and α' is a K-conjugate of α, then there exists a K-homomorphism (not necessarily unique) $\phi : L \to \mathbb{C}$ such that $\phi(\alpha) = \alpha'$.

   This problem did not originally specify "K-homomorphism," so it is okay if you do not prove that your homomorphism fixed K. On the other hand, you may assume in subsequent problems that you've proved the version of the problem stated above.

3. Recall that we called a field extension L/K *Galois* if, for every $\alpha \in L$, all K-conjugates $\alpha' \in \mathbb{C}$ of α are also in L. This definition can be unwieldy since there are infinitely many choices of α. Show that it suffices to check only α in a generating set. That is: if $L = K(\alpha_1, \cdots, \alpha_n)$, and L contains all K-conjugates of $\alpha_1, \cdots, \alpha_n$, then L is Galois.

4. Prove that a degree 2 field extension in $\mathbb{C}$ is always Galois.

5. We showed in class that $\mathbb{Q}(\sqrt[3]{2})/\mathbb{Q}$ is not Galois. Show that $\mathbb{Q}(\sqrt[3]{2}, \zeta_3)/\mathbb{Q}$, however, is Galois. What is the degree of that extension?