1. (Textbook 13.1 through 13.4)

(This sounds like a lot, but 13.1 is most of the work, and the others are fairly short. You should read the worked examples in Chapter 13 to see some applicable techniques.)

- 2. (Textbook 13.14)
- 3. As mentioned in class, $\mathbb{F}_2[t]/(t^3 + t^2 + 1) \cong \mathbb{F}_2[t]/(t^3 + t + 1)$, and both fields can be referred to as \mathbb{F}_8 . Exhibit an explicit bijection between these two quotients.
- 4. Construct a field of \mathbb{F}_{16} order 16. For each of the 16 elements, determine its \mathbb{F}_{2} -conjugates, degree over \mathbb{F}_2 , and minimal polynomial over \mathbb{F}_2 (try to find a way to present this information efficiently). Determine the Galois group of $\mathbb{F}_{16}/\mathbb{F}_2$ and all of its subgroups, and use this to list all subfields. For each subfield, explicitly list the elements.
- 5. Let p be an odd prime, and let L/\mathbb{F}_p be the splitting field of $t^4 + 1$, let ζ denote a root of $t^4 + 1$ in L, and let $\omega = \zeta + \zeta^{-1}$.
	- (a) Prove that $\omega^2 = 2$, and therefore that $t^2 2$ splits in \mathbb{F}_p if and only if $\omega \in \mathbb{F}_p$. We say that 2 is a quadratic residue mod p if $t^2 - 2$ splits in \mathbb{F}_p .
	- (b) Prove that $\omega^p =$ $\int \omega$ if $p \equiv \pm 1 \mod 8$ $-\omega$ if $p \equiv \pm 3 \mod 8$.
	- (c) Deduce that 2 is a quadratic residue modulo p if and only if $p \equiv \pm 1 \mod 8$.