- (Textbook 13.1 through 13.4)
  (This sounds like a lot, but 13.1 is most of the work, and the others are fairly short. You should read the worked examples in Chapter 13 to see some applicable techniques.)
- 2. (Textbook 13.14)
- 3. As mentioned in class,  $\mathbb{F}_2[t]/(t^3 + t^2 + 1) \cong \mathbb{F}_2[t]/(t^3 + t + 1)$ , and both fields can be referred to as  $\mathbb{F}_8$ . Exhibit an explicit bijection between these two quotients.
- 4. Construct a field of  $\mathbb{F}_{16}$  order 16. For each of the 16 elements, determine its  $\mathbb{F}_2$ -conjugates, degree over  $\mathbb{F}_2$ , and minimal polynomial over  $\mathbb{F}_2$  (try to find a way to present this information efficiently). Determine the Galois group of  $\mathbb{F}_{16}/\mathbb{F}_2$  and all of its subgroups, and use this to list all subfields. For each subfield, explicitly list the elements.
- 5. Let p be an odd prime, and let  $L/\mathbb{F}_p$  be the splitting field of  $t^4 + 1$ , let  $\zeta$  denote a root of  $t^4 + 1$  in L, and let  $\omega = \zeta + \zeta^{-1}$ .
  - (a) Prove that  $\omega^2 = 2$ , and therefore that  $t^2 2$  splits in  $\mathbb{F}_p$  if and only if  $\omega \in \mathbb{F}_p$ . We say that 2 is a *quadratic residue mod* p if  $t^2 2$  splits in  $\mathbb{F}_p$ .
  - (b) Prove that  $\omega^p = \begin{cases} \omega & \text{if } p \equiv \pm 1 \mod 8 \\ -\omega & \text{if } p \equiv \pm 3 \mod 8 \end{cases}$ .
  - (c) Deduce that 2 is a quadratic residue modulo p if and only if  $p \equiv \pm 1 \mod 8$ .