MATH 42
MIDTERM 1
20 FEBRUARY 2015

Name : _____

- The time limit is 50 minutes.
- No calculators or notes are permitted.
- Each problem is worth 5 points.

1	/5	**2**	/5
3	/5	**4**	/5
5	/5	**6**	/5
		$\sum$	/30

(1) Find all prime numbers p between 1 and 100 such that

$$p \equiv -1 \pmod{15}.$$

(2) Recall that a *primitive Pythagorean triple* consists of three positive integers (a, b, c) such that
   - $a^2 + b^2 = c^2$, and
   - there are no common factors of a, b and c.

   Find a primitive Pythagorean triple such that $a = 15$.

(3) Compute the greatest common divisor of 1106 and 203.

(4) Solve the following congruence.

$$28x \equiv 3 \pmod{149}$$

(5) Suppose that a, b, c are positive integers such that $\gcd(a, b) = 1$. Prove that if a divides bc, then a divides c.

(6) Suppose that you enter a store carrying a large supply of 6 dollar coins. The shopkeeper is able to make change using 28 dollar coins and 63 dollar coins. Find a way that you can purchase a 1 dollar item.

For partial credit, you may first assume that both you and the shopkeeper have a large supply of all three types of coins (6,28, and 63) and solve the problem in this context.

(additional space for work)