1. Let $S(n)$ denote the sum of the first n odd numbers (for example, $S(3) = 1 + 3 + 5 = 9$). Compute $S(1), S(2), S(3), S(4)$, and $S(5)$. What pattern do these numbers follow?

2. Find two pairs (a, b) of positive (in particular, nonzero) whole numbers satisfying the equation

$$a^2 - 2b^2 = 1.$$

3. A *prime number race* is held, as follows. There are two teams, called Team 1 and Team 2. The game consists of a sequence of rounds, numbered $1, 2, 3$, and so on. In round n:

   - If n is prime and leaves a remainder of 1 when divided by 3, Team 1 scores a point.
   - If n is prime and leaves a remainder of 2 when divided by 3, Team 2 scores a point.
   - If n is not prime, or n is divisible by 3, neither team scores a point (recall that 1 is not considered prime).

   Determine the score of the game after 120 rounds.

4. Prove that the numbers $\sqrt{3}$ and $\sqrt[3]{2}$ are both irrational.

5. Recall that a *primitive Pythagorean triple* consists of three positive integers (a, b, c) such that $a^2 + b^2 = c^2$, and there is no common factor of a, b, and c.

   (a) Find a primitive Pythagorean triple with $a = 33$.

   (b) Find a primitive Pythagorean triple with $c = 85$.

   (c) Find another possible answer to either part (a) or part (b).

   Note. In part (c), you answer should not simply swap the numbers a and b (for example, if $(3, 4, 5)$ were an answer to (a), then do not give $(4, 3, 5)$ for (c)).