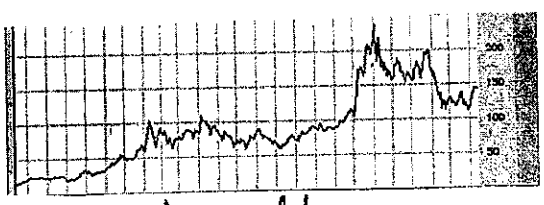
Worksheet for 29 October

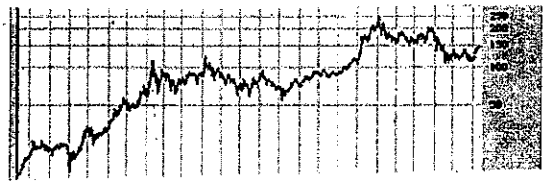
Logarithmic differentiation:

$$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}.$$

I may or may not discuss these.



linear plot



log. plot

In all these problems: don't try too hard to simplify the answer.

(1) Find $\frac{d}{dx}(x^x)$

(2) Find $\frac{d}{dx}\left((\cos x)^{\sin x}\right)$

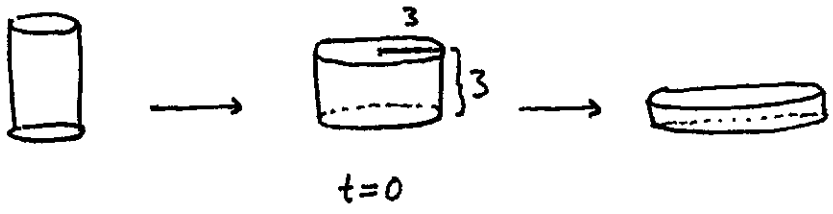
(3) Find $\frac{d}{dx}\left(x^{\sqrt{x}} + 7^x\right)$

(4) Find $\frac{d}{dx}\left[(\ln x + x)^{\sqrt{x}}\right]$

(5) Find $\frac{d}{dx}\left[\frac{(x+2)^{27}}{e^x\sqrt{x^2+167}}\right]$

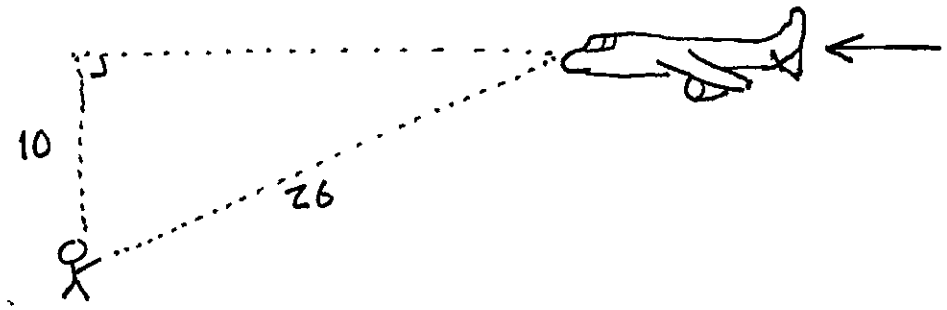
Part II : related rates

① A disc of hot metal flattens overtime, but is always shaped like a cylinder. The volume does not change.



At $t=0$ seconds, it is 3cm tall and 3cm in radius. The height is currently falling at 0.1 cm/sec. How fast is the radius increasing?

(2)

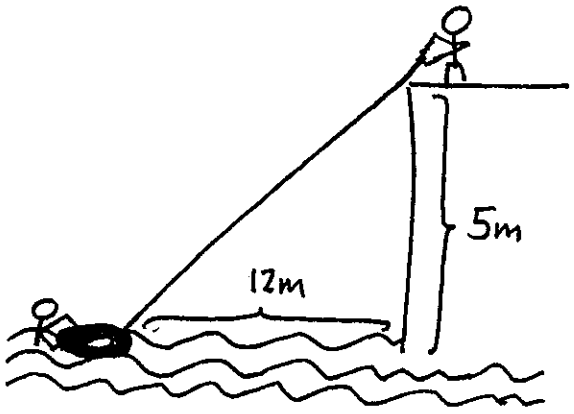


You are standing below the flight path of an airplane cruising at an altitude of 10km. With a radar device, you measure that the plane is 26km from you, and its distance to you is decreasing at a rate of 840km/hour.

a) What is the speed of the airplane?

b) How quickly is the angle your arm makes with the ground increasing, as you track the airplane?

③



(not to scale as usual)

You are pulling an overboard passenger back to the boat. The passenger is 12m from the boat being drawn in by 1m each second. How quickly are you drawing in the rope to do this?

④ An ideal gas obeys the equation $PV = nRT$

(P = pressure, V = volume, n = # moles of molecules, R = constant, T = temp.)

T must be measured in absolute terms (Kelvin).

Suppose:

$P = 10^5$ Pa
$V = 1$ m^3
$T = 300$K (27° C)

$V' = 0.1$ m^3/sec
$T' = 3$ K/sec

Find P' at this moment.