

Worksheet for 11/7/13

Part 1: compute the following limits:

$$\textcircled{1} \lim_{x \rightarrow \infty} (x \cdot e^{-x}) = \lim_{\infty \cdot 0} \frac{x}{e^x} \stackrel{\text{l'h\^op}}{=} \lim_{\frac{\infty}{\infty}} \frac{1}{e^x} = \boxed{0}$$

$$\textcircled{2} \lim_{x \rightarrow \infty} (x \cdot \sin(\frac{3}{x})) = \lim_{\infty \cdot 0} \frac{\sin(3/x)}{1/x} = \lim \frac{\cos(3/x) \cdot (-3/x^2)}{-1/x^2} \\ = \lim \frac{3 \cos(3/x)}{1} = \boxed{3}$$

$$\textcircled{3} \text{ (Pset 9, A6)} \lim_{x \rightarrow 0^+} (1+x)^{1/x} = e^{\lim_{1^{\infty}} \frac{1}{x} \ln(1+x)} = e^{\lim \frac{1/(1+x)}{1}} = e^1 = \boxed{e}$$

$$\textcircled{4} \lim_{x \rightarrow \infty} (x^{1/x}) = e^{\lim_{\infty^0} \frac{1}{x} \ln x} = e^{\lim \frac{1/x}{1/x^2}} = e^0 = \boxed{1}$$

$$\textcircled{5} \lim_{x \rightarrow 0^+} (x^x) = e^{\lim_{0^0} x \ln x} = e^{\lim \frac{\ln x}{1/x}} = e^{\lim \frac{1/x}{-1/x^2}} = e^{\lim(-x)} = \boxed{1}$$

$$\textcircled{6} \lim_{x \rightarrow 0^+} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{\infty - \infty} \frac{x - \sin x}{x \sin x} = \lim_{\frac{0}{0}} \frac{1 - \cos x}{\sin x + x \cos x} = \lim \frac{\sin x}{\cos x + \cos x - x \sin x} = \boxed{0}$$

~~OR = $\frac{1 - (\sin x)^2}{\sin x}$~~

$$\textcircled{7} \lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) = \lim_{\infty - \infty} \frac{x}{\sqrt{x^2 + x} + x} = \lim \frac{1}{\sqrt{1 + 1/x} + 1} = \boxed{\frac{1}{2}}$$

Part II

- ⑧ An object in free fall on the moon accelerates downward at about 1.6 m/s^2 (about 17% of Earth's gravity).

If you jump off a 5m tall ledge on the moon, how long will it take to hit the bottom? How fast will you be falling when you land?

$$a(t) = -1.6$$

$$\text{so } v(t) = -1.6t + C$$

$$v(0) = 0, \text{ so } C = 0.$$

$$v(t) = -1.6t$$

$$h(t) = -0.8t^2 + 5$$

$$h(t) = 0$$

$$\Leftrightarrow 0.8t^2 = 5$$

$$\Leftrightarrow t^2 = \frac{25}{4}$$

$$\Leftrightarrow t = 2.5$$

$$= 5/2$$

$$\boxed{2.5 \text{ seconds}}$$

$$v(2.5) = -1.6 \cdot 2.5$$

$$= \boxed{4 \text{ m/s}}$$

- ⑨ Find some antiderivative for each function.

a) x^7
 $\frac{1}{8}x^8$

e) $x + \cos x$
 $\frac{1}{2}x^2 + \sin x$

b) 3^x
 $\frac{1}{\ln 3} \cdot 3^x$

f) e^{7x}
 $\frac{1}{7}e^{7x}$

c) $2\cos x$
 $2\sin x$

g) $\frac{x+x^7}{x^2} = \frac{1}{x} + x^5$
 $-\ln|x| + \frac{1}{6}x^6$

d) $x^2 + 7$
 $\frac{1}{3}x^3 + 7x$

h) $x \cdot e^x$ (this one is a challenge...
be creative)

$$\underline{x \cdot e^x - e^x}$$